STANDARD PARALLELISM

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Standard C++ Library Evolution Chair, US Programming Languages Chair

Conventions

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```
namespace stdv = std::views;

namespace stdr = std::ranges;

namespace ex = std::execution;

namespace this_thread = std::this_thread;
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namespace stdr = std::ranges;

namespace ex = std::execution;

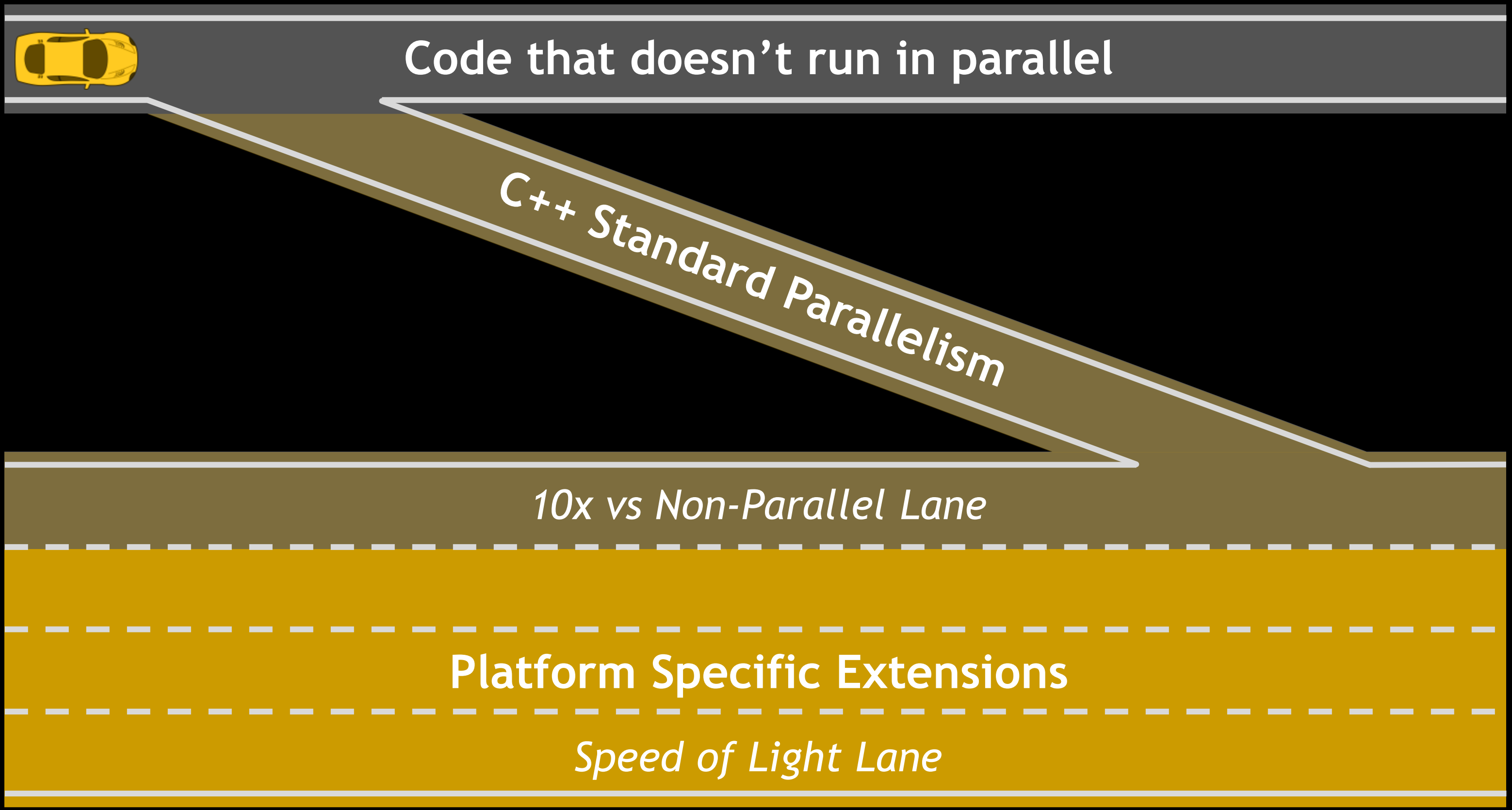
namespace this_thread = std::this_thread;
```

Class Template Argument Deduction (CTAD)

```
std::tuple t{3.14, 42}; ⟶ std::tuple<double, int>

std::array a{0, 1, 1, 0}; ⟶ std::array<int, 4>
```

We Need On-Ramps



What Makes C++ Portable?

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Non-8-bit char
Noncommittal sizeof
Non-2's comp int
Non-IEEE float
Non-endian pointers
Aligned addressing
Segmented memory

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	Important in the 20th century?
Non-8-bit char	☑
Noncommittal sizeof	☑
Non-2's comp int	☑
Non-IEEE float	☑
Non-endian pointers	☑
Aligned addressing	☑
Segmented memory	☑

What Makes C++ Portable?

	Important in the 20th century?	Important in the 21st century?
Non-8-bit `char`	☑	☒
Noncommittal `sizeof`	☑	☒
Non-2's comp `int`	☑	☒
Non-IEEE `float`	☑	☒
Non-endian pointers	☑	☒
Aligned addressing	☑	☑
Segmented memory	☑	☑

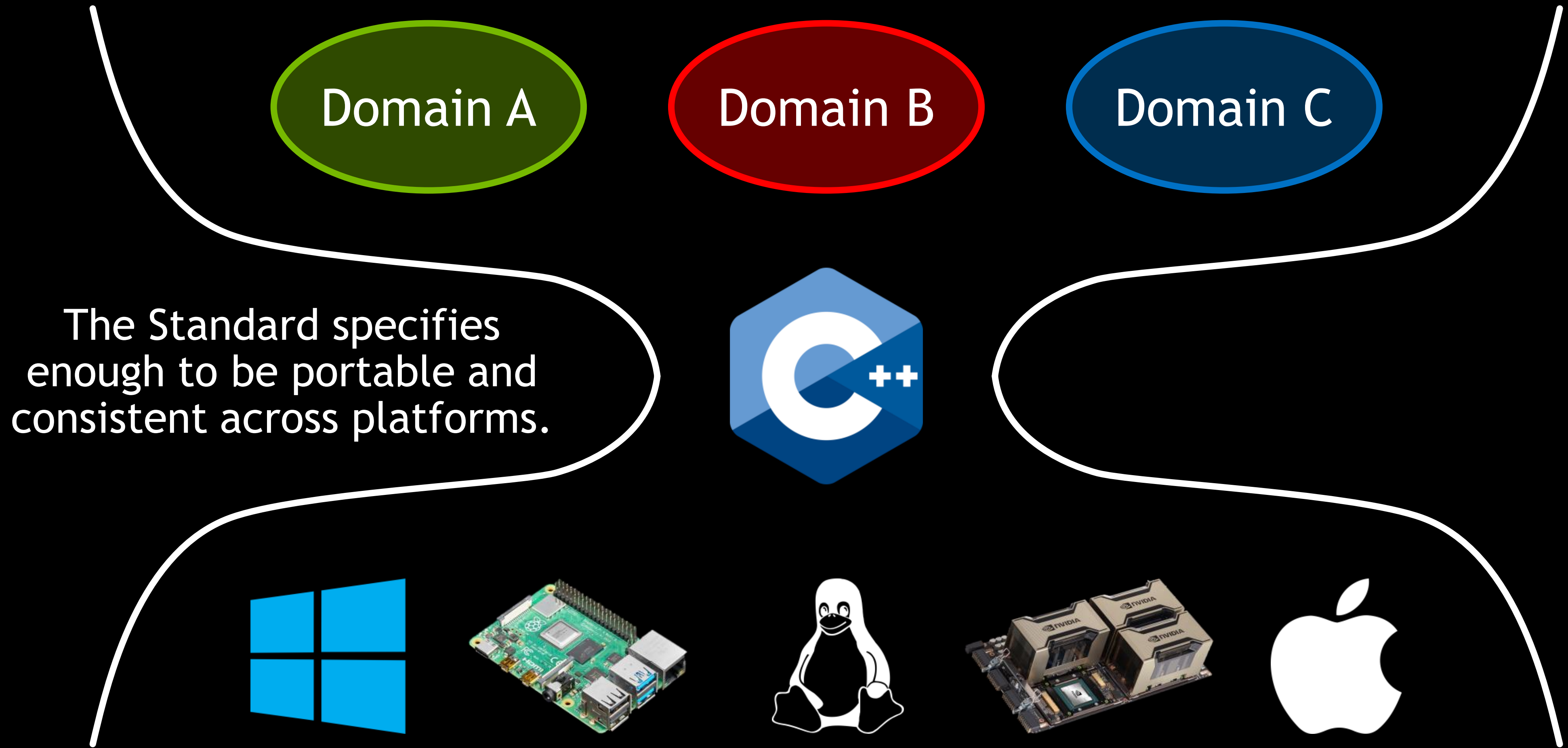
The New Portability Contract

- Memory Model
- Execution Model
- Forward Progress Guarantees
- Concurrency Primitives

The New Portability Contract

- Memory Model
- Execution Model
- Forward Progress Guarantees
- Concurrency Primitives
- *Parallelism Primitives*
- *Asynchrony Model*

The C++ Standard is *descriptive*, not *prescriptive*.


Domain A
Domain B
Domain C
The Standard specifies enough to be portable and consistent across platforms.

Domain A

Domain B

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The Standard grants enough freedom for each platform to choose the right design.

Implementation Freedom

Domain A

Domain B

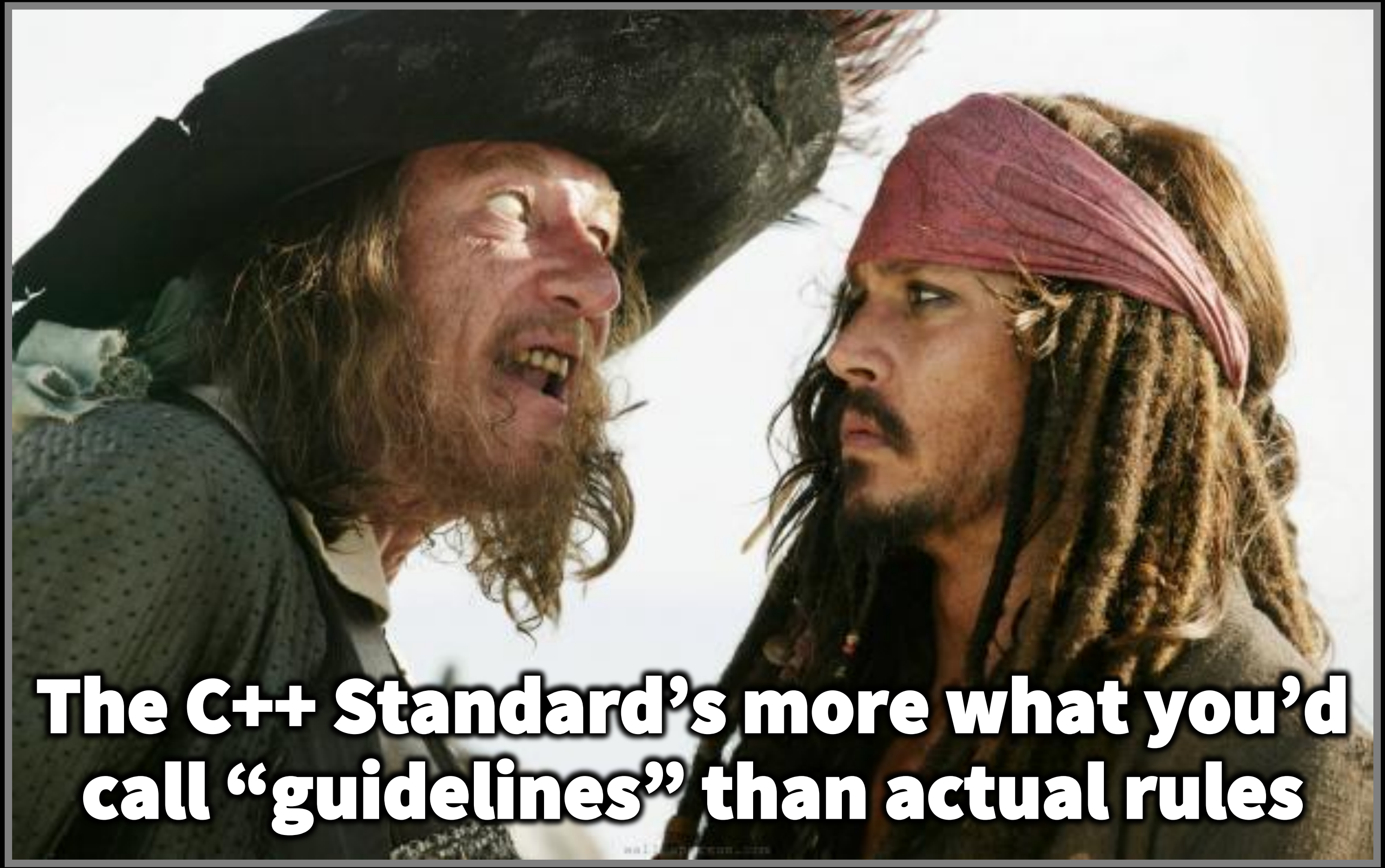
Domain C

The Standard specifies enough to be portable and consistent across platforms.

The Standard grants enough freedom for each platform to choose the right design.

Implementation-defined and undefined behavior are often a feature, not a bug.

Implementation Freedom



Standard Parallelism

is in the Library

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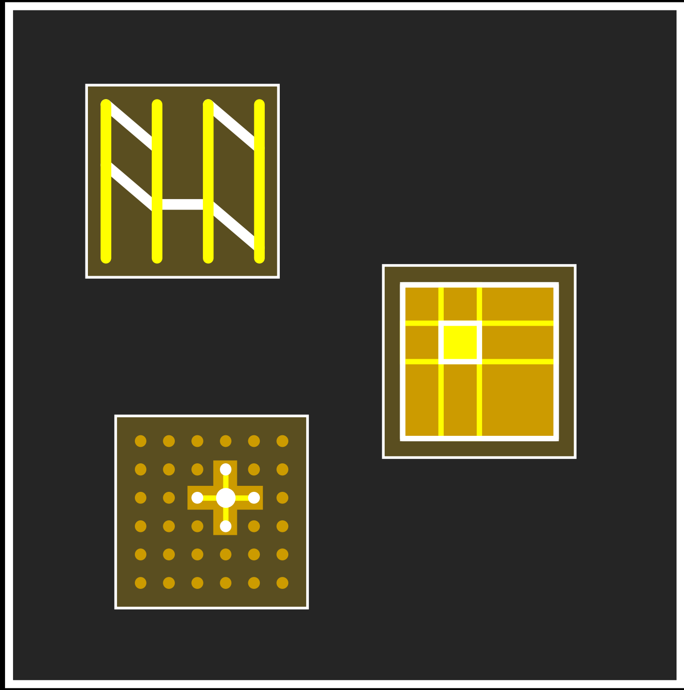
Pillars of C++ Standard Parallelism

Common Algorithms that Dispatch to Vendor-Optimized Parallel Libraries



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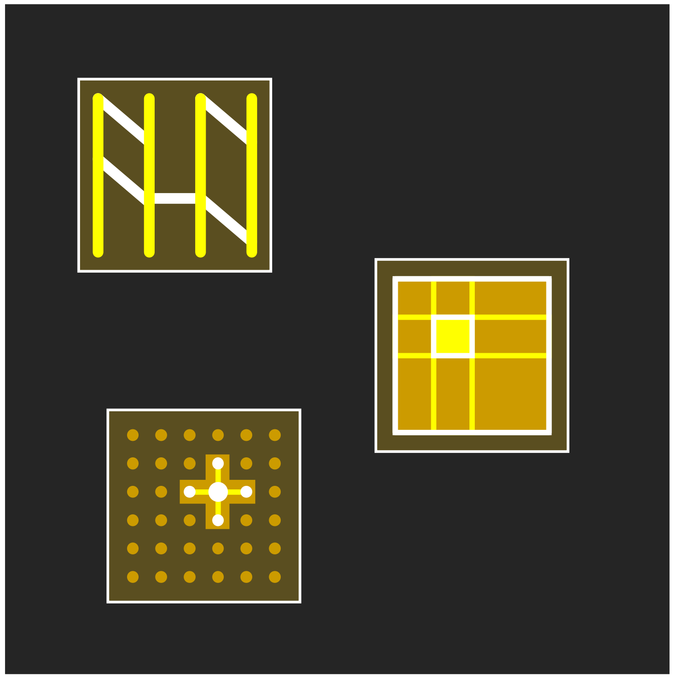
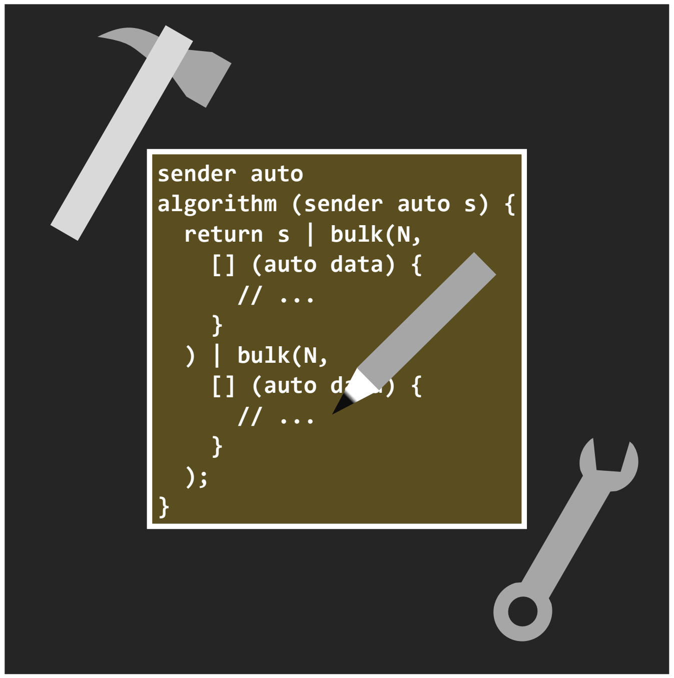


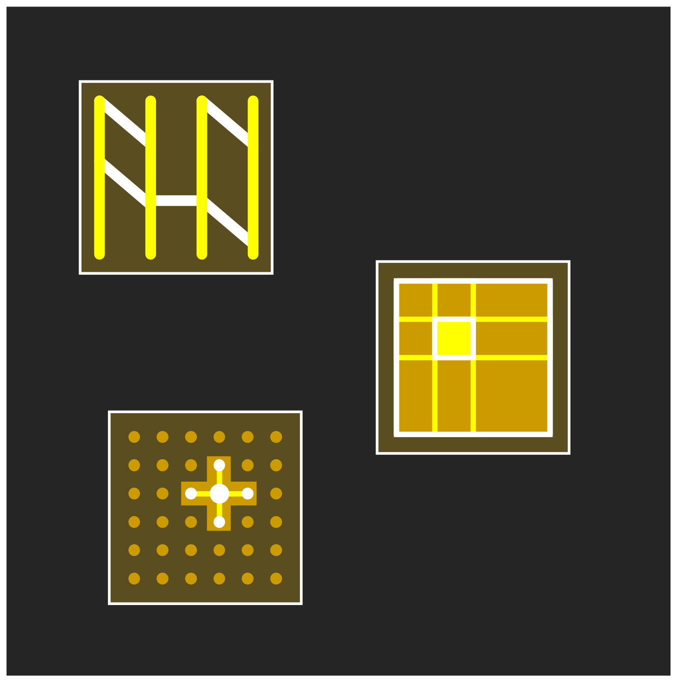
Tools to Write Your Own Parallel Algorithms that Run Anywhere

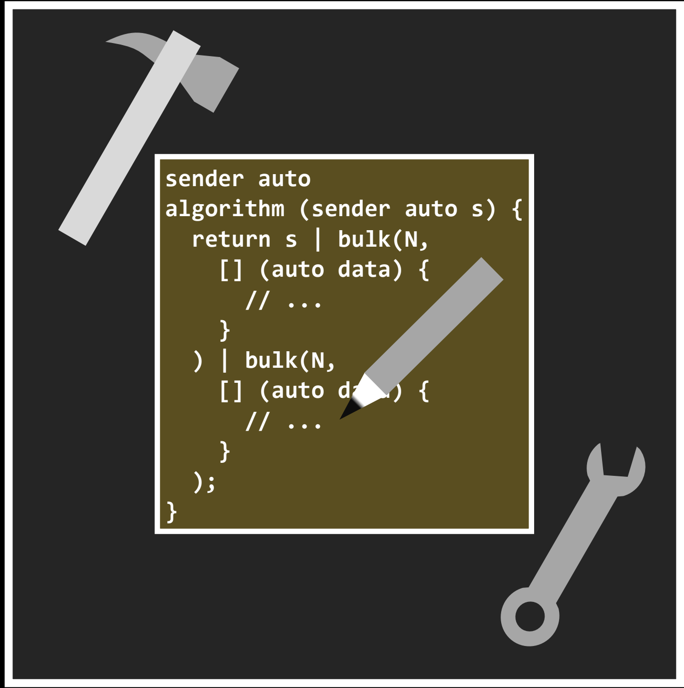


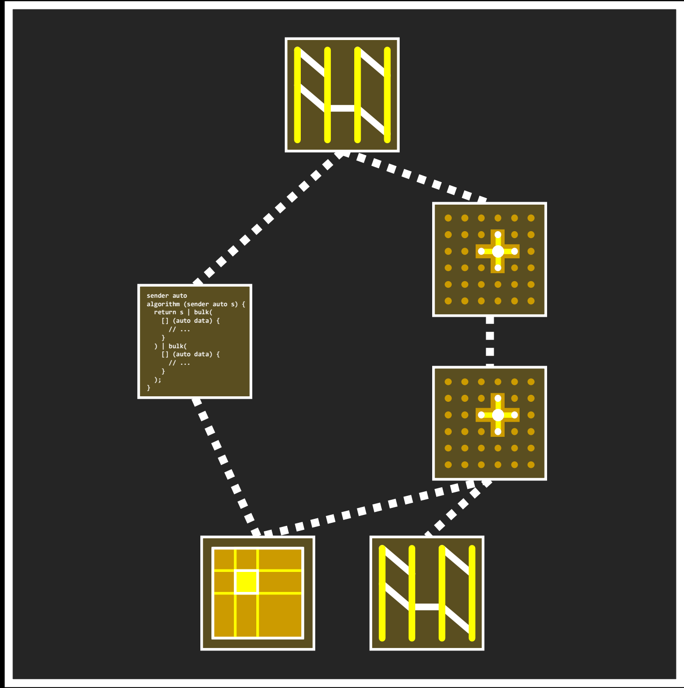
```
sender auto
algorithm (sender auto s) {
  return s | bulk(N,
    [] (auto data) {
      // ...
    }
  ) | bulk(N,
    [] (auto data) {
      // ...
    }
  );
}
```

Pillars of C++ Standard Parallelism

Common Algorithms that Dispatch to Vendor-Optimized Parallel Libraries	Tools to Write Your Own Parallel Algorithms that Run Anywhere	Mechanisms for Composing Parallel Invocations into Task Graphs







Standard Algorithms

Serial (C++98)

```
std::vector<T> x{…};

std::for_each(
 begin(x), end(x),
 f);

std::for_each(
 begin(x), end(x),
 g);

std::for_each(
 begin(x), end(x),
 h);
```

```
std::vector<double> x{…}, y{…};
double dot_product = std::transform_reduce(begin(x), end(x),
                                           begin(y));

std::span<std::string_view> s{…};
std::sort(begin(s), end(s));

std::unordered_map<std::string_view, int> db{…};
std::vector<std::pair<std::string_view, int>> m{…};
std::copy_if(begin(db), end(db), begin(m),
             [] (auto e) { return e.second > 0; });
```

Standard Algorithms

adjacent_difference	is_sorted[_until]	rotate[_copy]
adjacent_find	lexicographical_compare	search[_n]
all_of	max_element	set_difference
any_of	merge	set_intersection
copy[_if|_n]	min_element	set_symmetric_difference
count[_if]	minmax_element	set_union
equal	mismatch	sort
fill[_n]	move	stable_partition
find[_end|_first_of|_if|_if_not]	none_of	stable_sort
for_each	nth_element	swap_ranges
generate[_n]	partial_sort[_copy]	transform
includes	partition[_copy]	uninitialized_copy[_n]
inplace_merge	remove[_copy|_copy_if|_if]	uninitialized_fill[_n]
is_heap[_until]	replace[_copy|_copy_if|_if]	unique
is_partitioned	reverse[_copy]	unique_copy

Standard Algorithms

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 begin(x), end(x),
 h);
```

Parallel (C++17)

```
std::vector<T> x{…};

std::for_each(
 ex::par_unseq,
 begin(x), end(x),
 f);
std::for_each(
 ex::par_unseq,
 begin(x), end(x),
 g);
std::for_each(
 ex::par_unseq,
 begin(x), end(x),
 h);
```

```
std::vector<double> x{…}, y{…};
double dot_product = std::transform_reduce(ex::par_unseq,
                                           begin(x), end(x),
                                           begin(y));

std::span<std::string_view> s{…};
std::sort(ex::par_unseq, begin(s), end(s));

std::unordered_map<std::string_view, int> db{…};
std::vector<std::pair<std::string_view, int>> m{…};
std::copy_if(ex::par_unseq, begin(db), end(db), begin(m),
             [] (auto e) { return e.second > 0; });
```

Execution Policy	Operations occur …	Operations are …

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`std::execution::par`	Potentially in multiple threads	Indeterminately sequenced within each thread
`std::execution::par_unseq`	Potentially in multiple threads	Unsequenced

```
std::size_t word_count(std::string_view s) {
  …
}
```

```
std::string_view frost = "Whose woods these are I think I know.\n"
                         "His house is in the village though;  \n"
                         "He will not see me stopping here     \n"
                         "To watch his woods fill up with snow.\n";
word_count(frost);
```

```cpp
std::size_t word_count(std::string_view s) {
  if (s.empty()) return 0;
  return std::transform_reduce(ex::par_unseq, …);
}
```

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```

```
std::size_t word_count(std::string_view s) {
  if (s.empty()) return 0;
  return std::transform_reduce(ex::par_unseq,
    begin(s), end(s) - 1, begin(s) + 1,
    …
  );
}
```

```
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std::size_t word_count(std::string_view s) {
  if (s.empty()) return 0;
  return std::transform_reduce(ex::par_unseq,
    begin(s), end(s) - 1, begin(s) + 1,
    …

    [] (char l, char r) { return std::isspace(l) && !std::isspace(r); }
  );
}

std::string_view frost = "Whose woods these are I think I know.\n"
                         "His house is in the village though;  \n"
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    begin(s), end(s) - 1, begin(s) + 1,
    …

    [] (char l, char r) { return std::isspace(l) && !std::isspace(r); }
  );
}

std::size_t result     =  0000010000010000010001010000010100000
                         10001000001001001000100000001000000000
                         10010000100010001001000000001000000000
                         10010000010001000001000010010000100000;
```

#include <C++>

```
std::size_t word_count(std::string_view s) {
  if (s.empty()) return 0;
  return std::transform_reduce(ex::par_unseq,
    begin(s), end(s) - 1, begin(s) + 1,
    std::size_t(!std::isspace(s.front()) ? 1 : 0),
    …
    [] (char l, char r) { return std::isspace(l) && !std::isspace(r); }
  );
}

std::size_t result     = 10000010000010000010001010000010100000
                         10001000001001001000100000001000000000
                         10010000100010001001000000001000000000
                         10010000010001000001000010010000100000;
```

```
std::size_t word_count(std::string_view s) {
  if (s.empty()) return 0;
  return std::transform_reduce(ex::par_unseq,
    begin(s), end(s) - 1, begin(s) + 1,
    std::size_t(!std::isspace(s.front()) ? 1 : 0),
    std::plus(),
    [] (char l, char r) { return std::isspace(l) && !std::isspace(r); }
  );
}

std::size_t result     = 1  +  1  +  1  +  1 + 1+1  +  1+1 +
                         1 + 1  +  1 +1 +1 + 1   +   1     +
                         1 +1  + 1 + 1 + 1 +1   +    1     +
                         1 +1  +  1 + 1  +  1  + 1 +1  + 1 ;
```

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std::size_t word_count(std::string_view s) {
  if (s.empty()) return 0;
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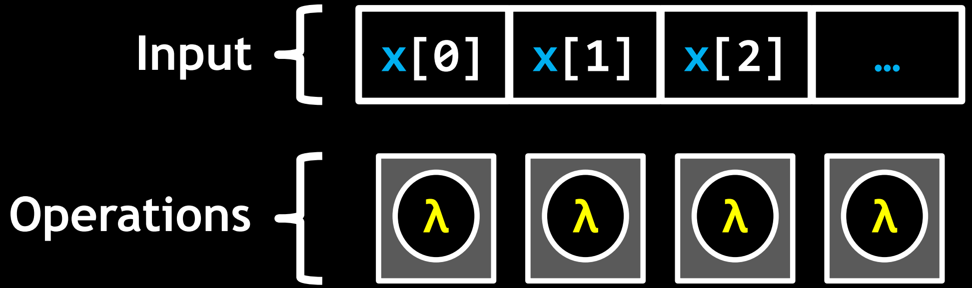
word_count(frost);
```

In C++20, the Standard Library introduced ranges.

Unlike iterators, ranges are composable and can be lazy.

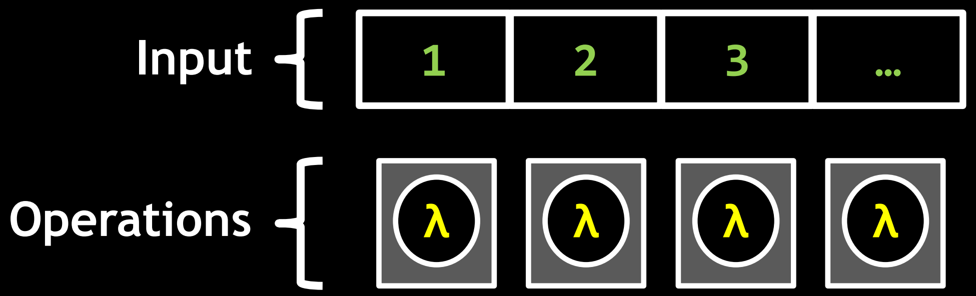
```
std::vector x{…};

std::for_each(
  ex::par_unseq,
  begin(x), end(x),
  […] (auto& obj) { … });
```



```
auto v = stdv::iota(1, N);

std::for_each(
  ex::par_unseq,
  begin(v), end(v),
  […] (auto idx) { … });
```



```
std::span A{input,  N * M};
std::span B{output, M * N};

auto v = stdv::cartesian_product(
  stdv::iota(0, N),
  stdv::iota(0, M));

std::for_each(ex::par_unseq,
  begin(v), end(v),
  [=] (auto idx) {
    auto [i, j] = idx;
    B[i + j * N] = A[i * M + j];
  });
```

#include <C++>

```
std::span A{input,  N * M};
std::span B{output, M * N};

auto v = stdv::cartesian_product(
  stdv::iota(0, N),
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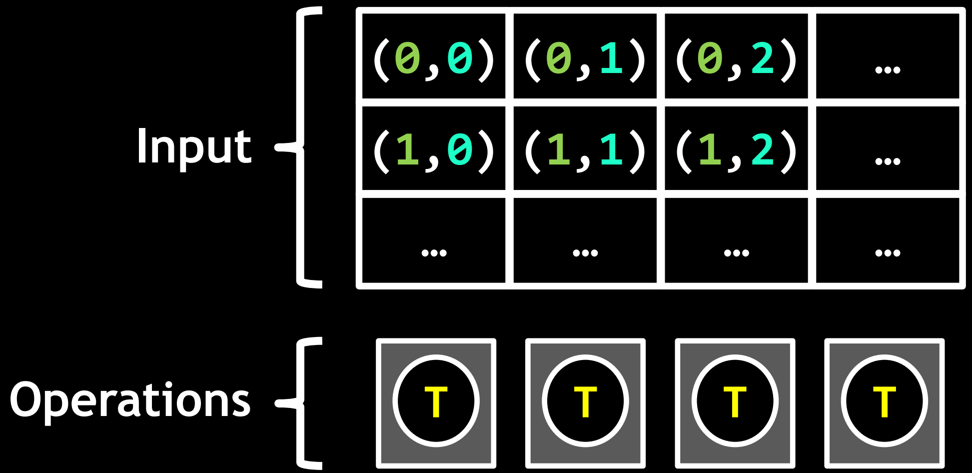
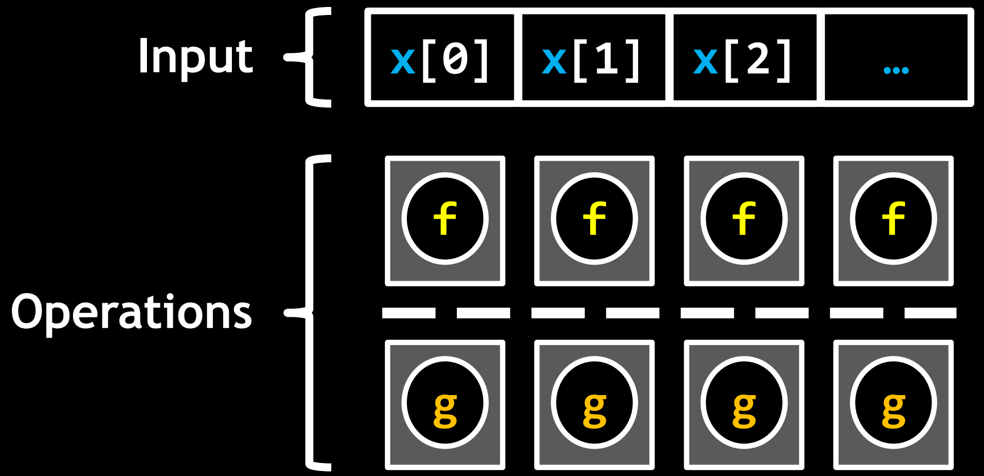


```
std::vector x{…};

std::for_each(ex::par_unseq,
              begin(x), end(x), f);
std::for_each(ex::par_unseq,
              begin(x), end(x), g);
```



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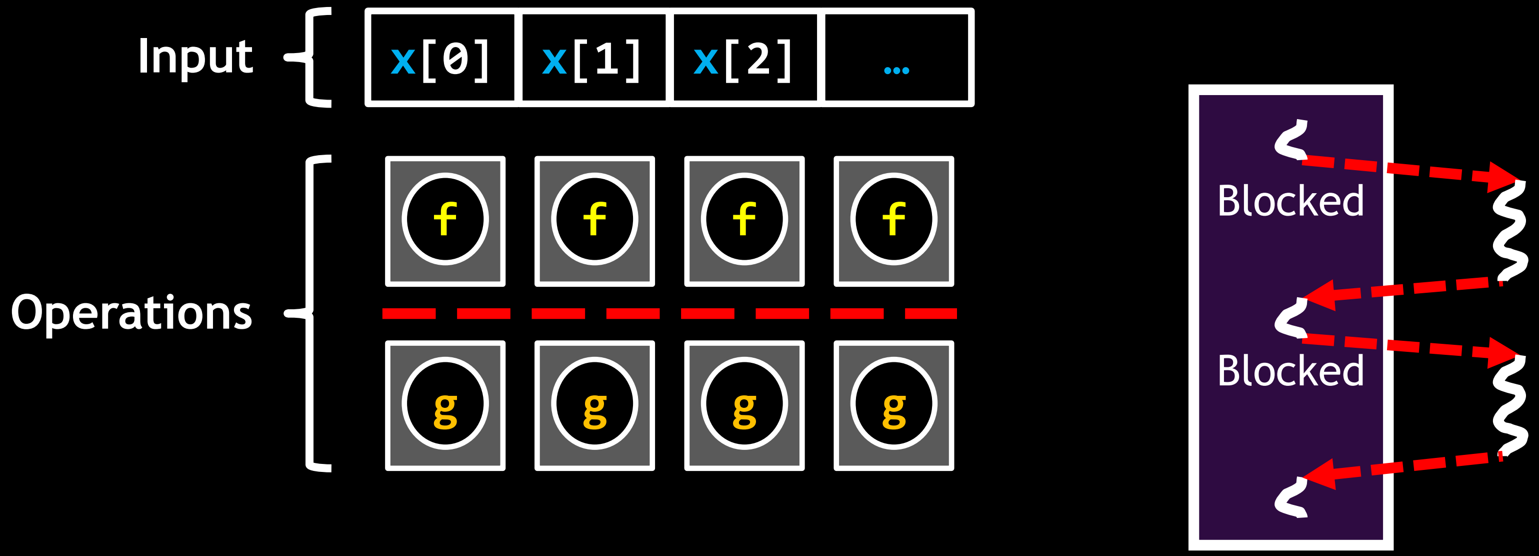
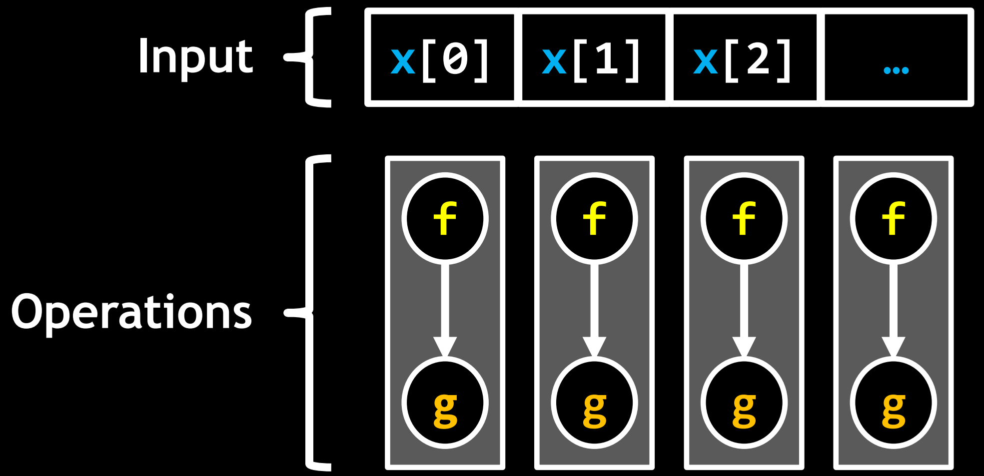


```
std::vector x{…};

auto v = stdv::transform(x, f);

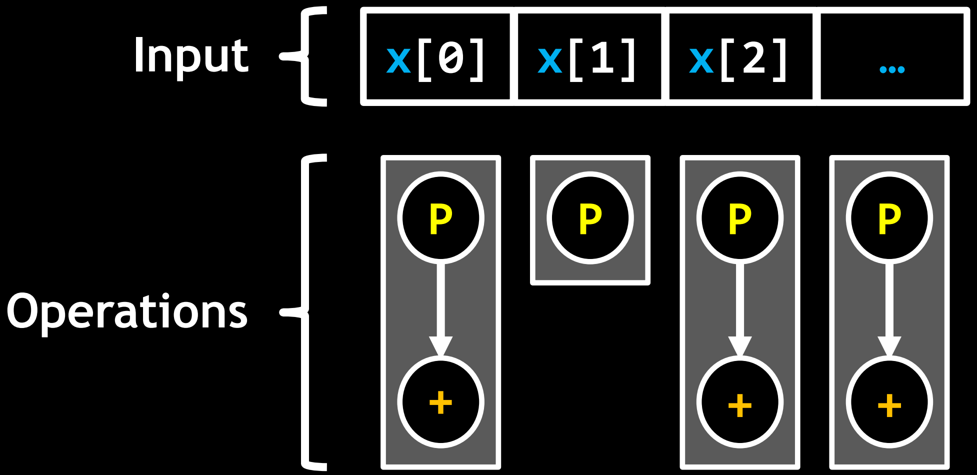
std::for_each(ex::par_unseq,
              begin(v), end(v), g);
```



```
std::vector x{…};

auto v = stdv::filter(x,
  [] (auto e) { return e > 0; });

std::reduce(ex::par_unseq,
            begin(v), end(v));
```



The C++ parallel algorithms introduced in C++17 are great,

but they’re just the start of the story.

```
std::vector<std::string_view> s{…};

std::sort(ex::par_unseq, begin(s), end(s));
std::unique(ex::par_unseq, begin(s), end(s));
```

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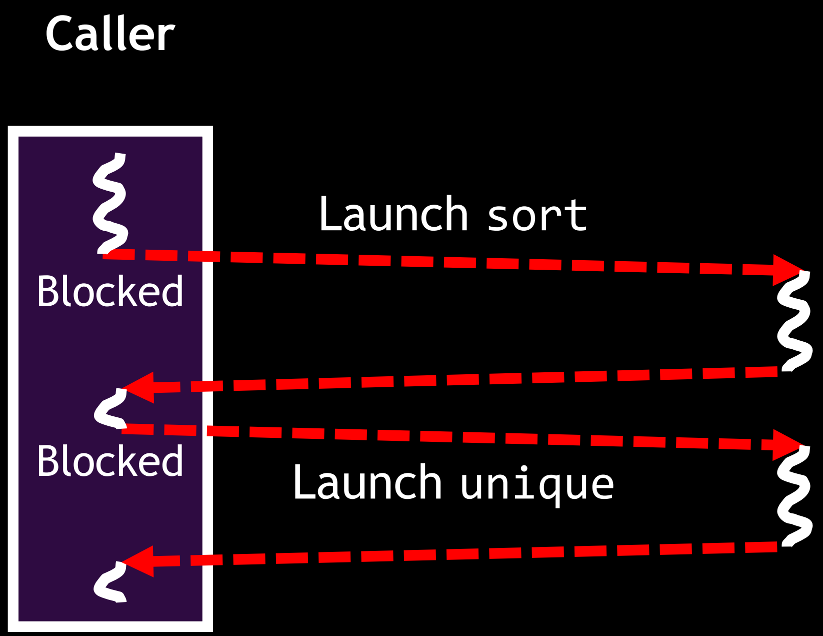

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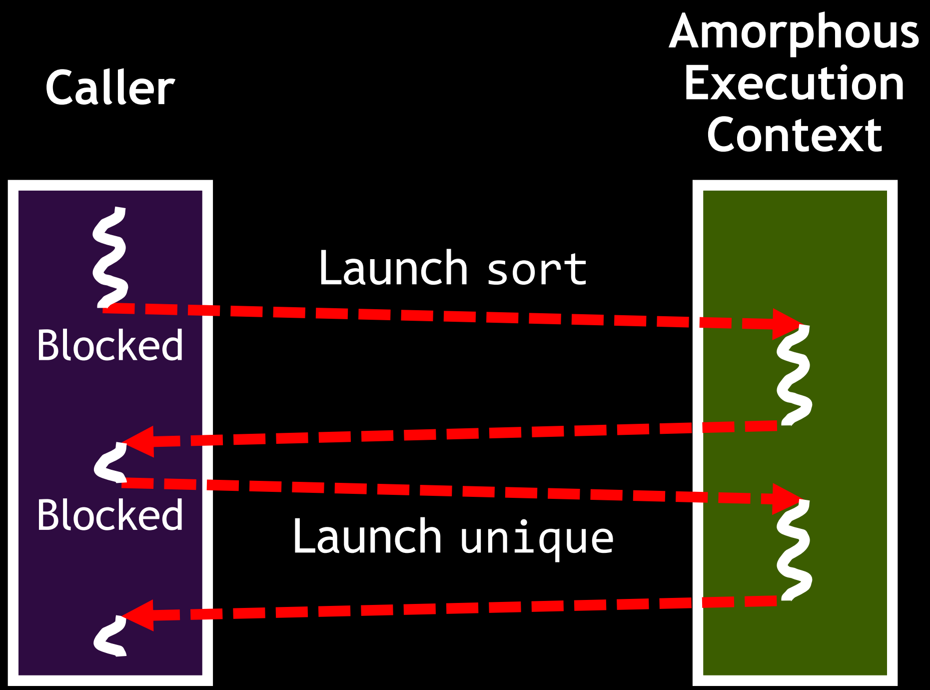

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- No standard way to express where things should execute.

The solution is coming soon:

Senders & Receivers

```
ex::scheduler auto sch = thread_pool.scheduler();

ex::sender auto begin = ex::schedule(sch);
ex::sender auto hi    = ex::then(begin, [] { return 13; });
ex::sender auto add   = ex::then(hi, [] (int a) { return a + 42; });

auto [i] = this_thread::sync_wait(add).value();
```

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ex::scheduler auto sch = thread_pool.scheduler();

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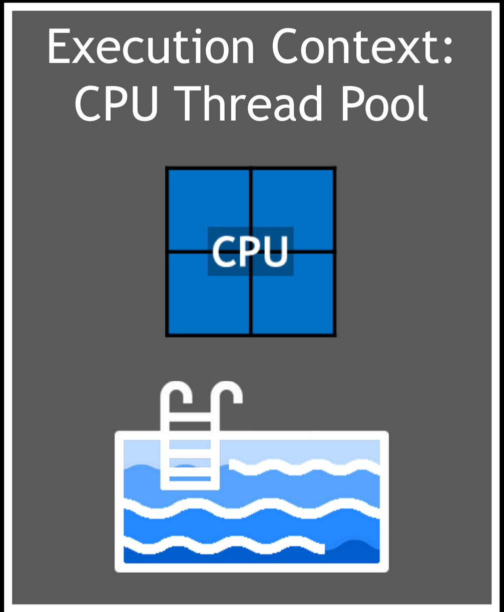
Receivers process asynchronous signals.

Schedulers are handles to execution contexts.


Execution Context:
CPU Thread Pool
CPU

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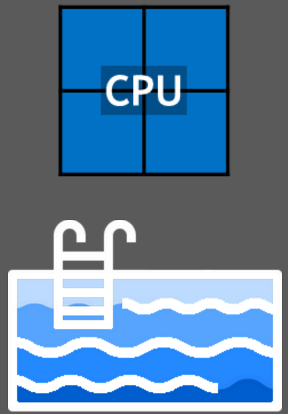
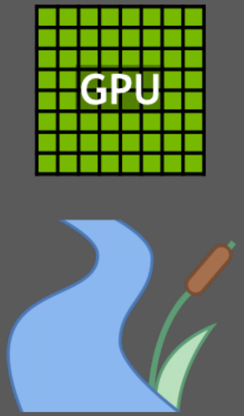
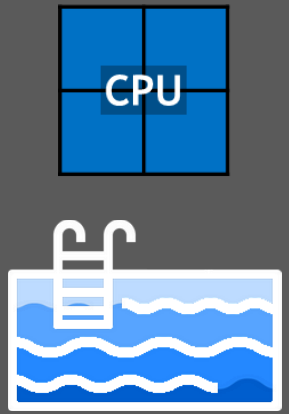


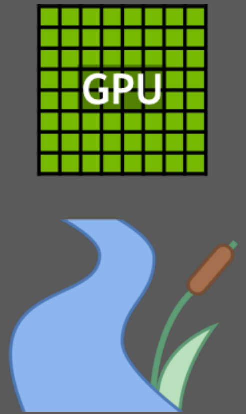
Execution Context:
GPU Stream



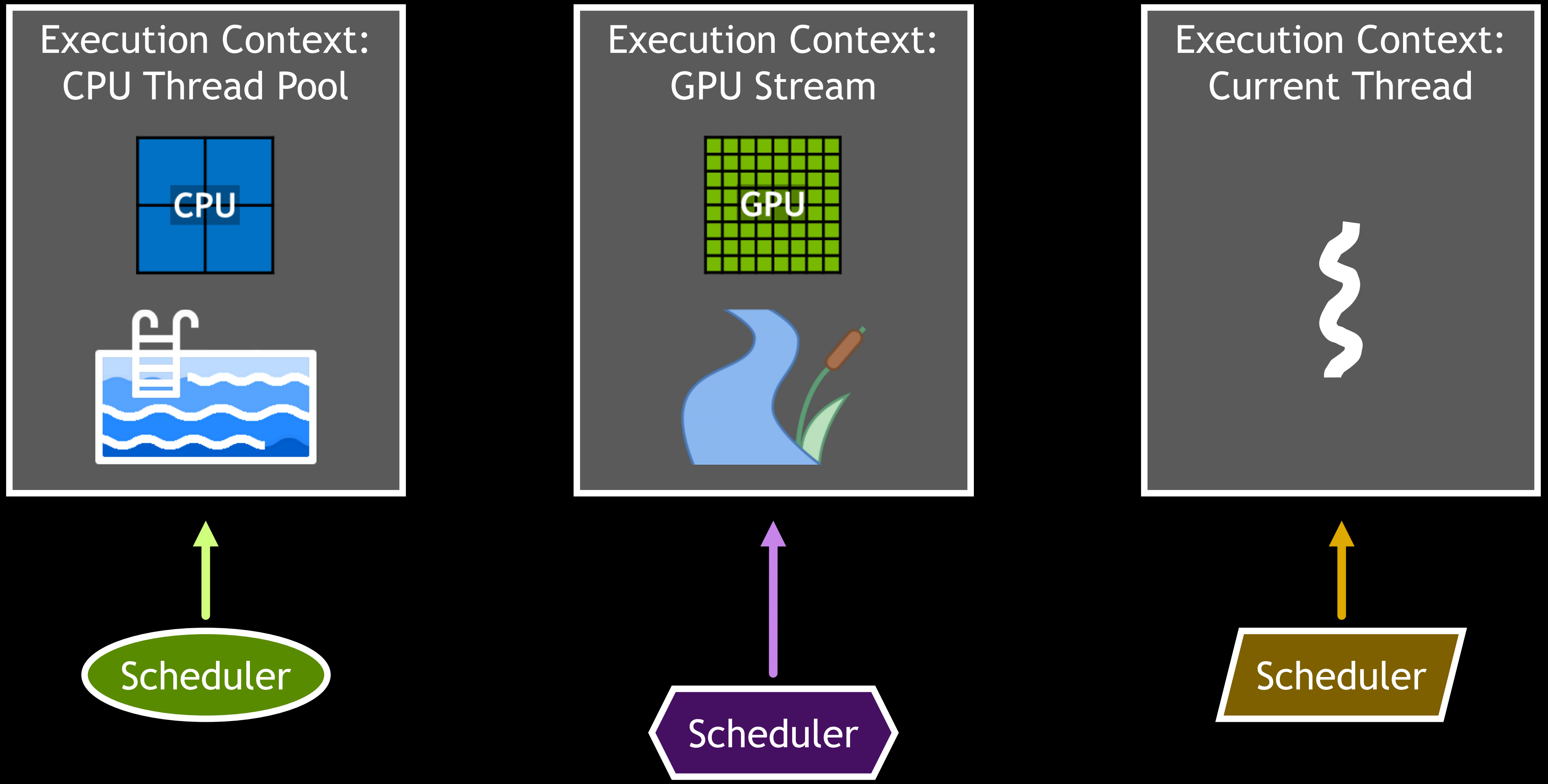
Execution Context: CPU Thread Pool

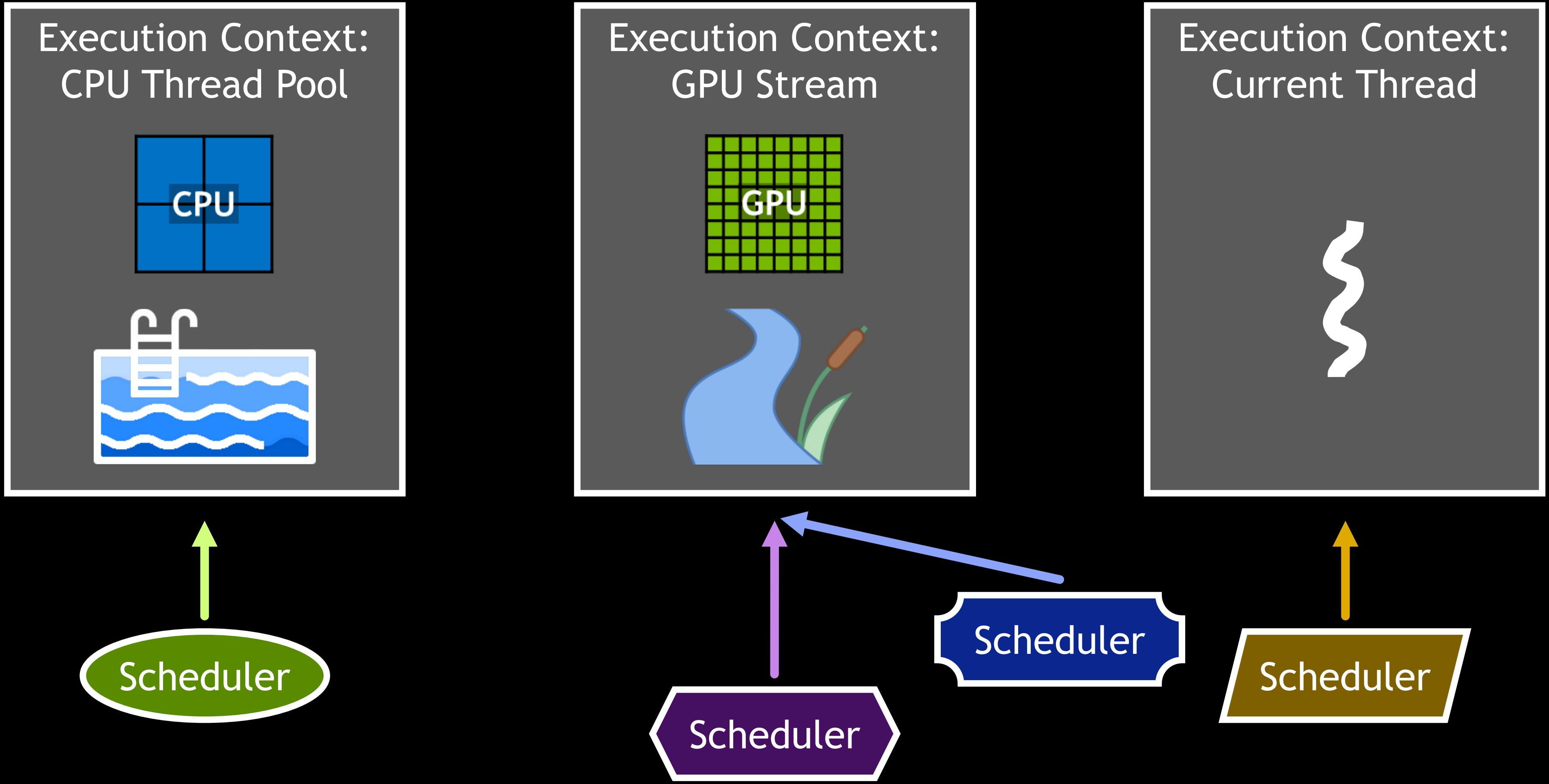


Execution Context: GPU Stream



Execution Context: Current Thread


Execution Context:
CPU Thread Pool
CPU
Execution Context:
GPU Stream
GPU
Execution Context:
Current Thread
Scheduler
Scheduler
Scheduler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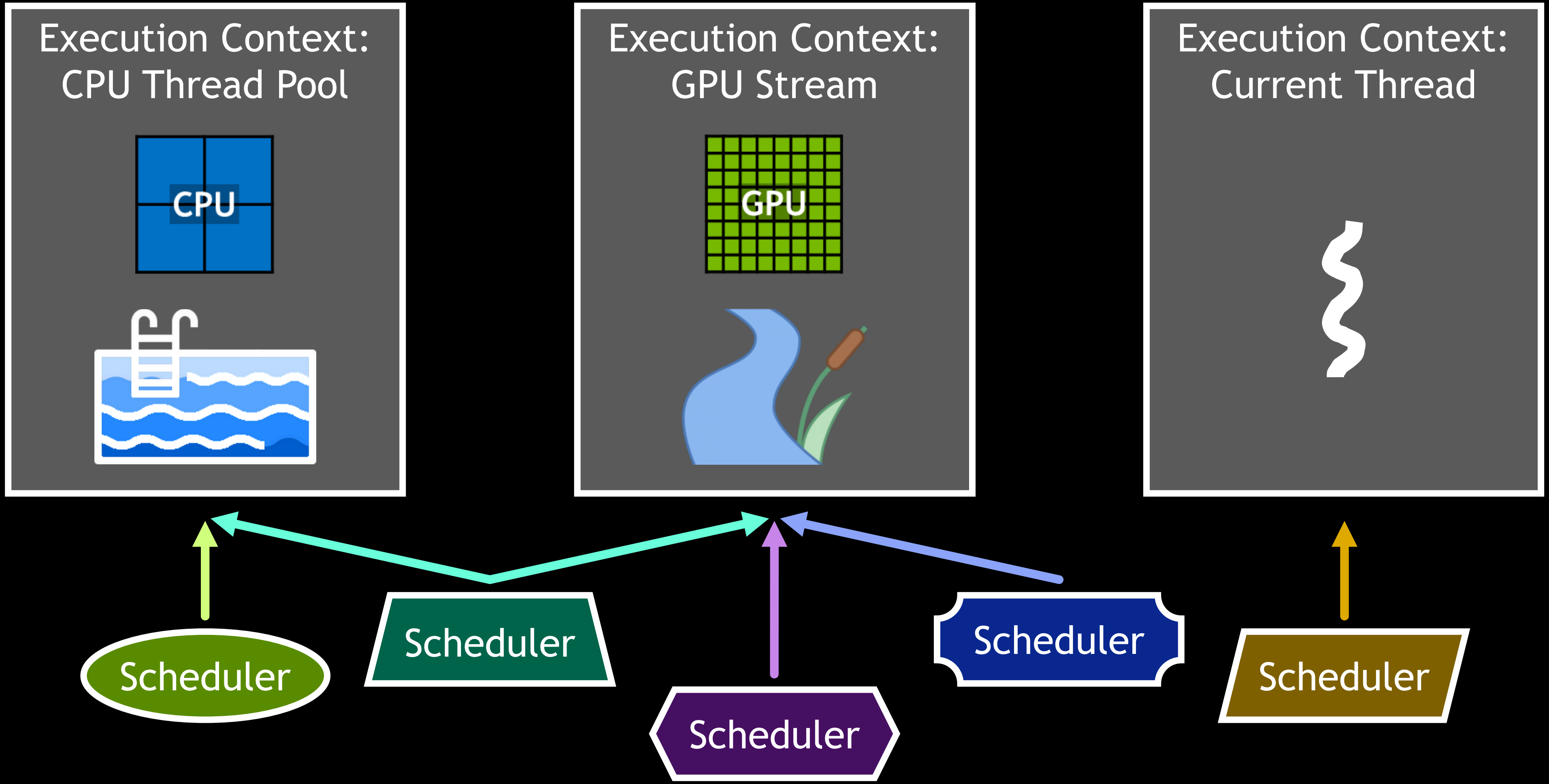
Scheduler
Scheduler


Execution Context:
CPU Thread Pool
CPU
Execution Context:
GPU Stream
GPU
Execution Context:
Current Thread
Scheduler
Scheduler
Scheduler
Scheduler
Scheduler

Schedulers produce senders.

- Senders represent asynchronous work.

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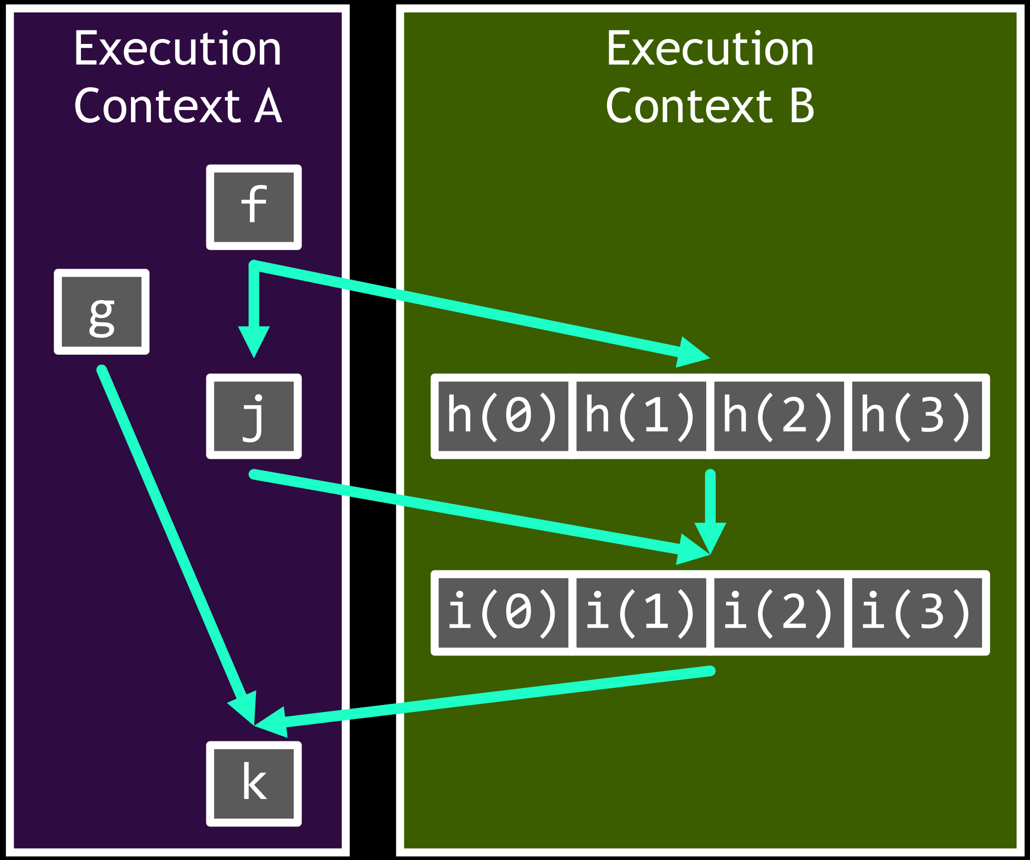
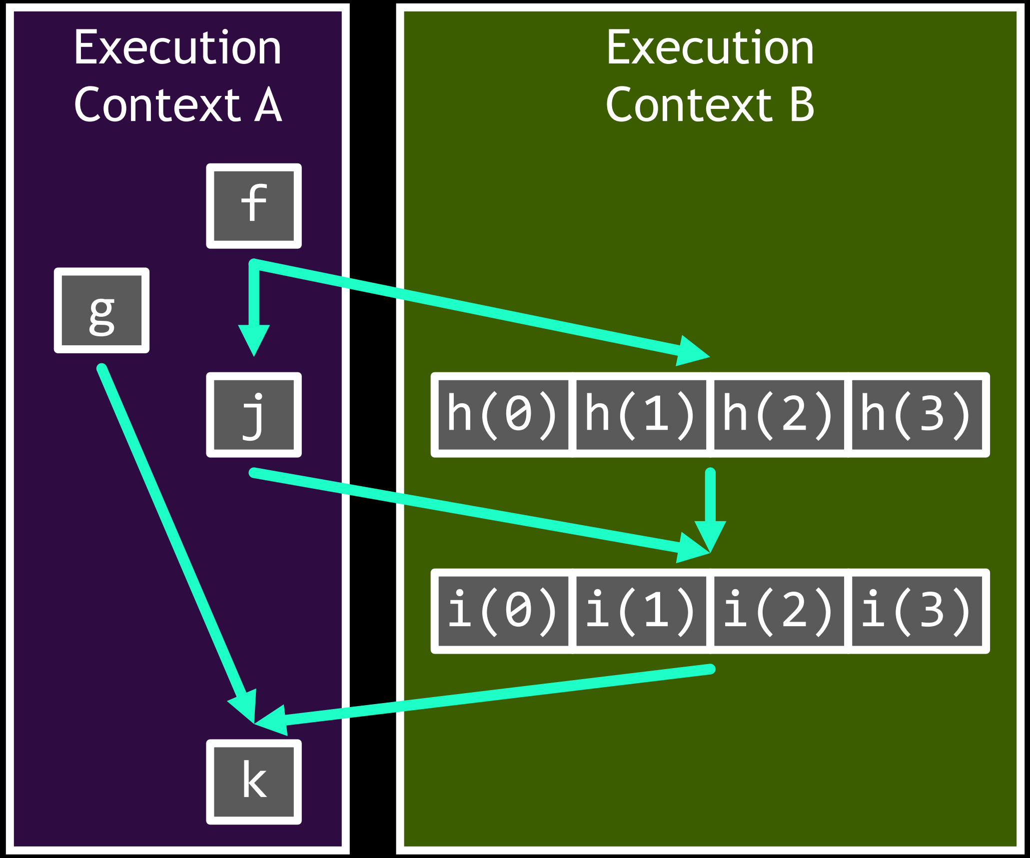
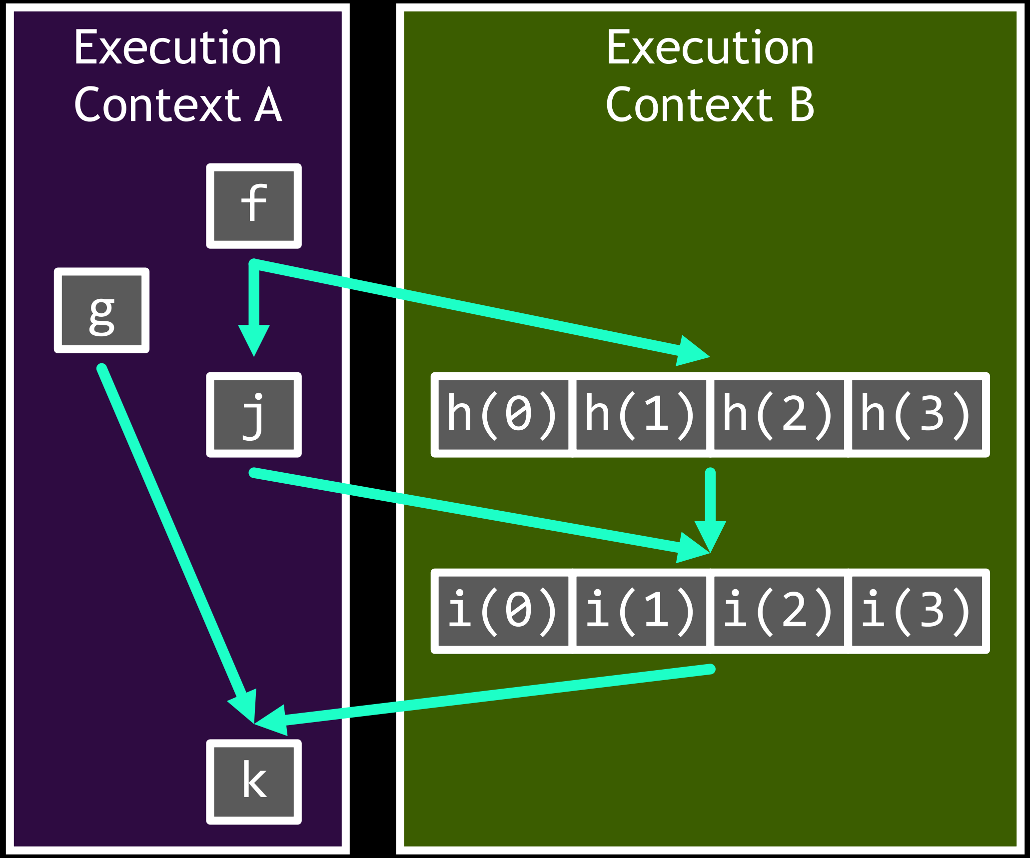
- Senders form the nodes of a task graph.



- Senders represent asynchronous work.
- Senders form the nodes of a task graph.
- Senders are lazy.

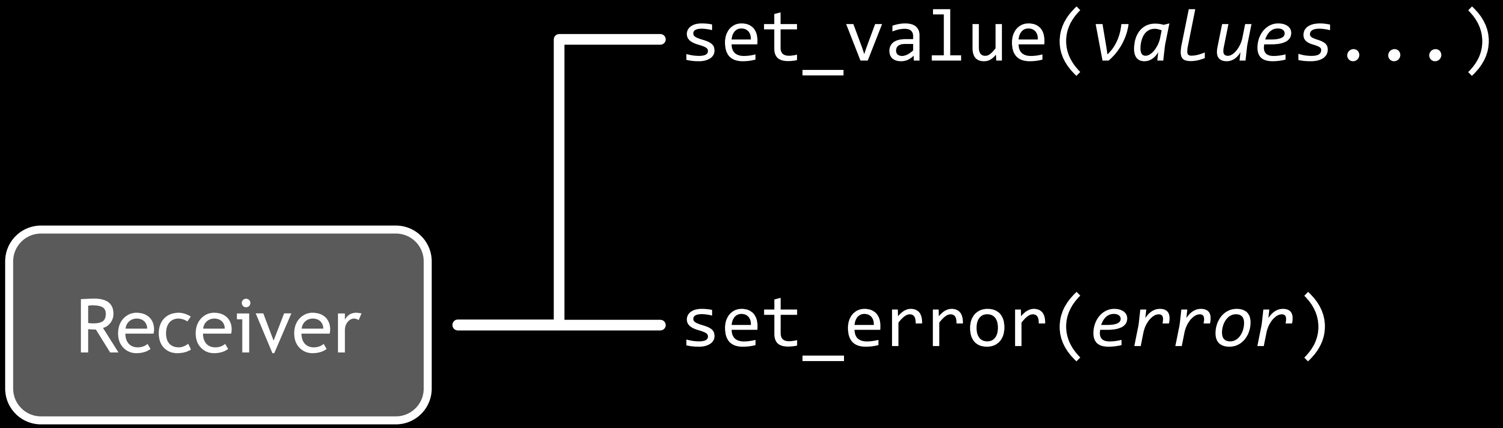


- Senders represent asynchronous work.
- Senders form the nodes of a task graph.
- Senders are lazy.
- When a sender's work completes, it sends a signal to the receivers attached to it.

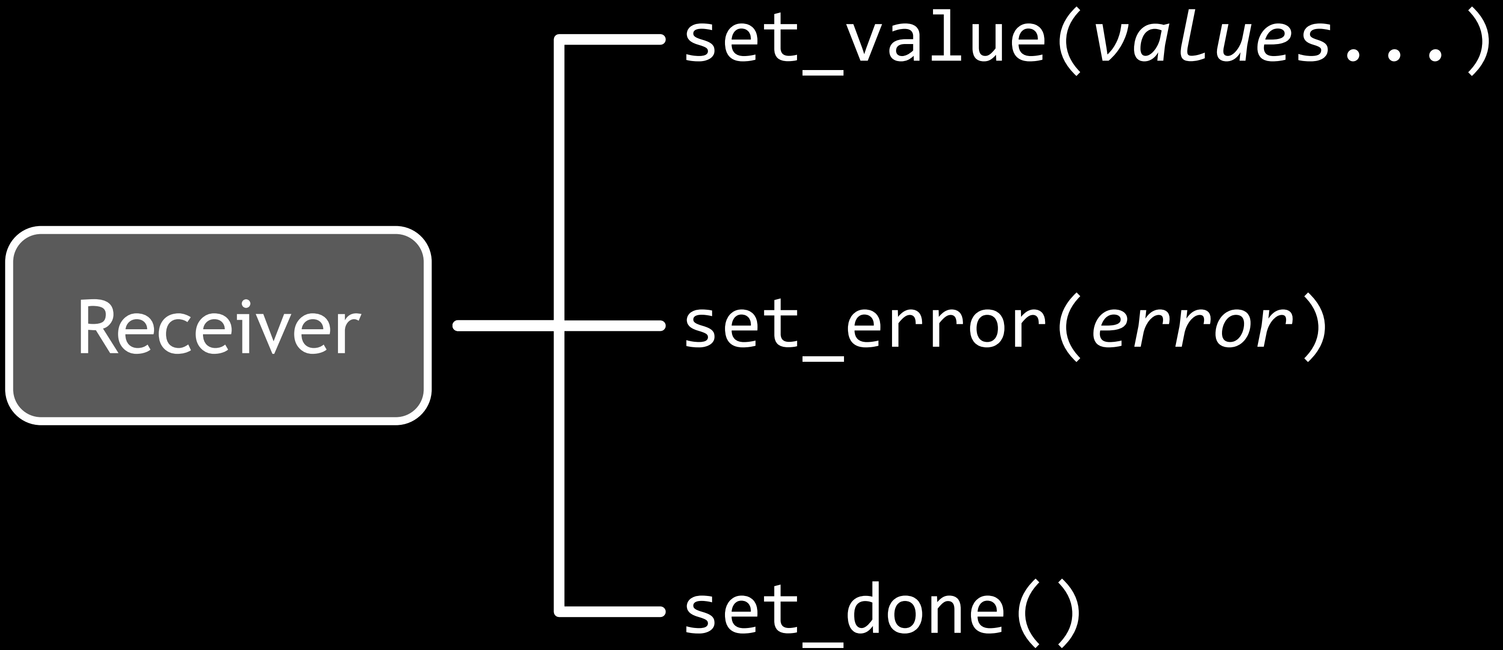


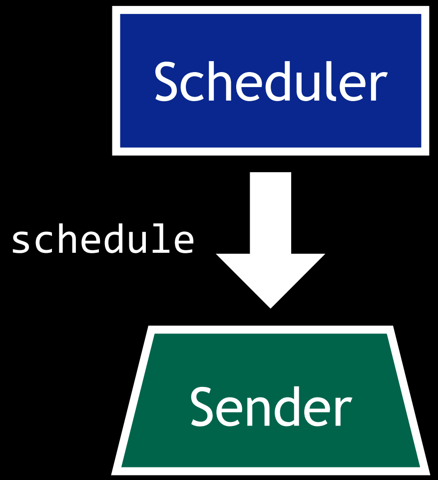
Receiver

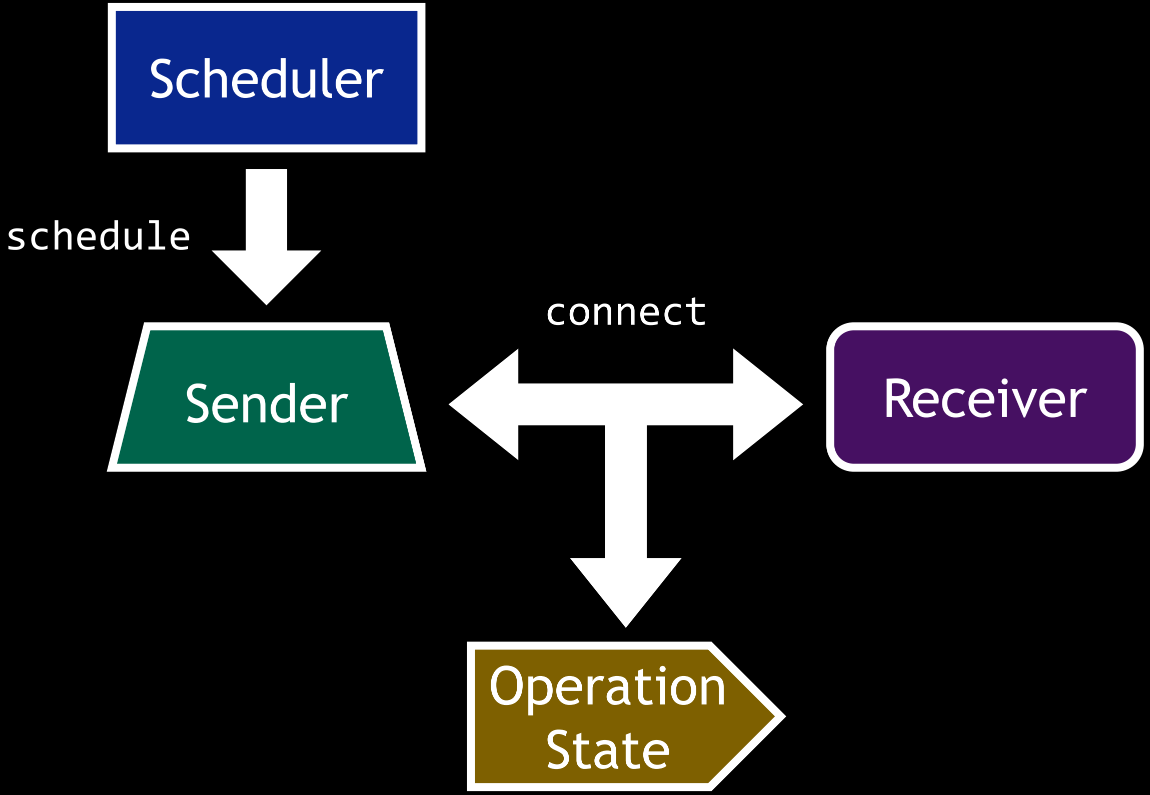

set_value(values...)
Receiver

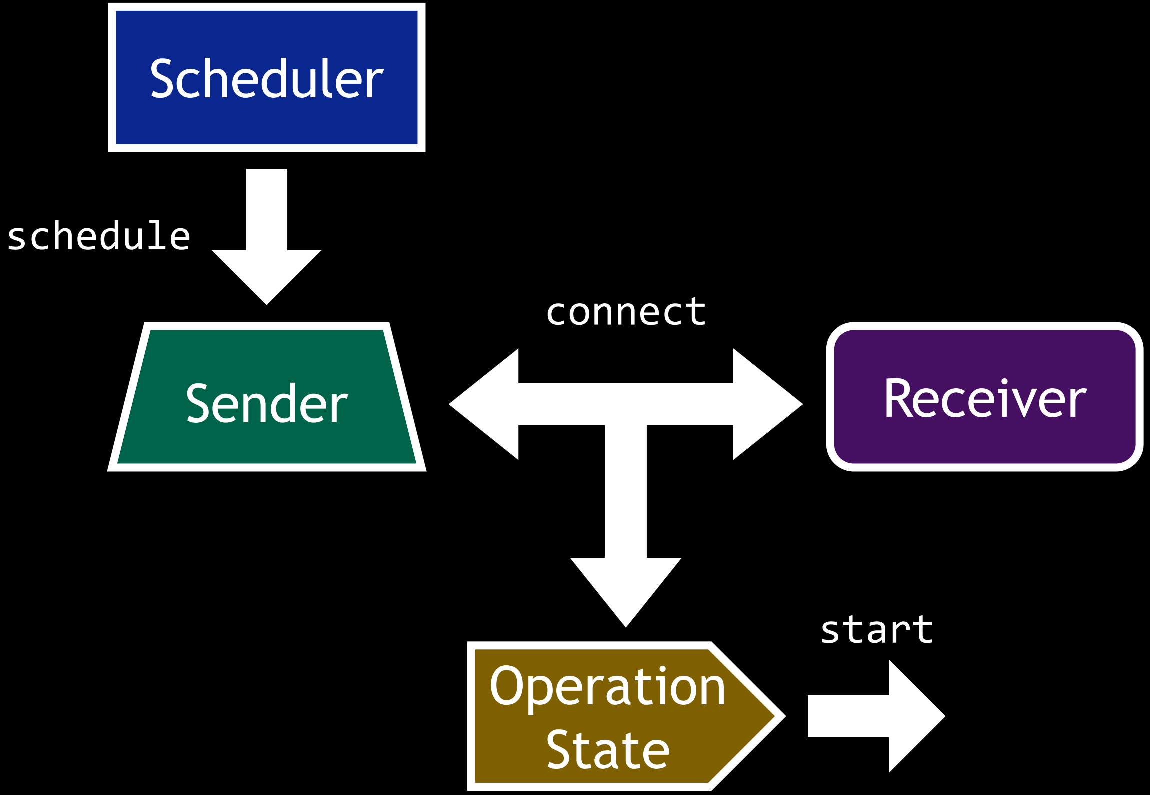


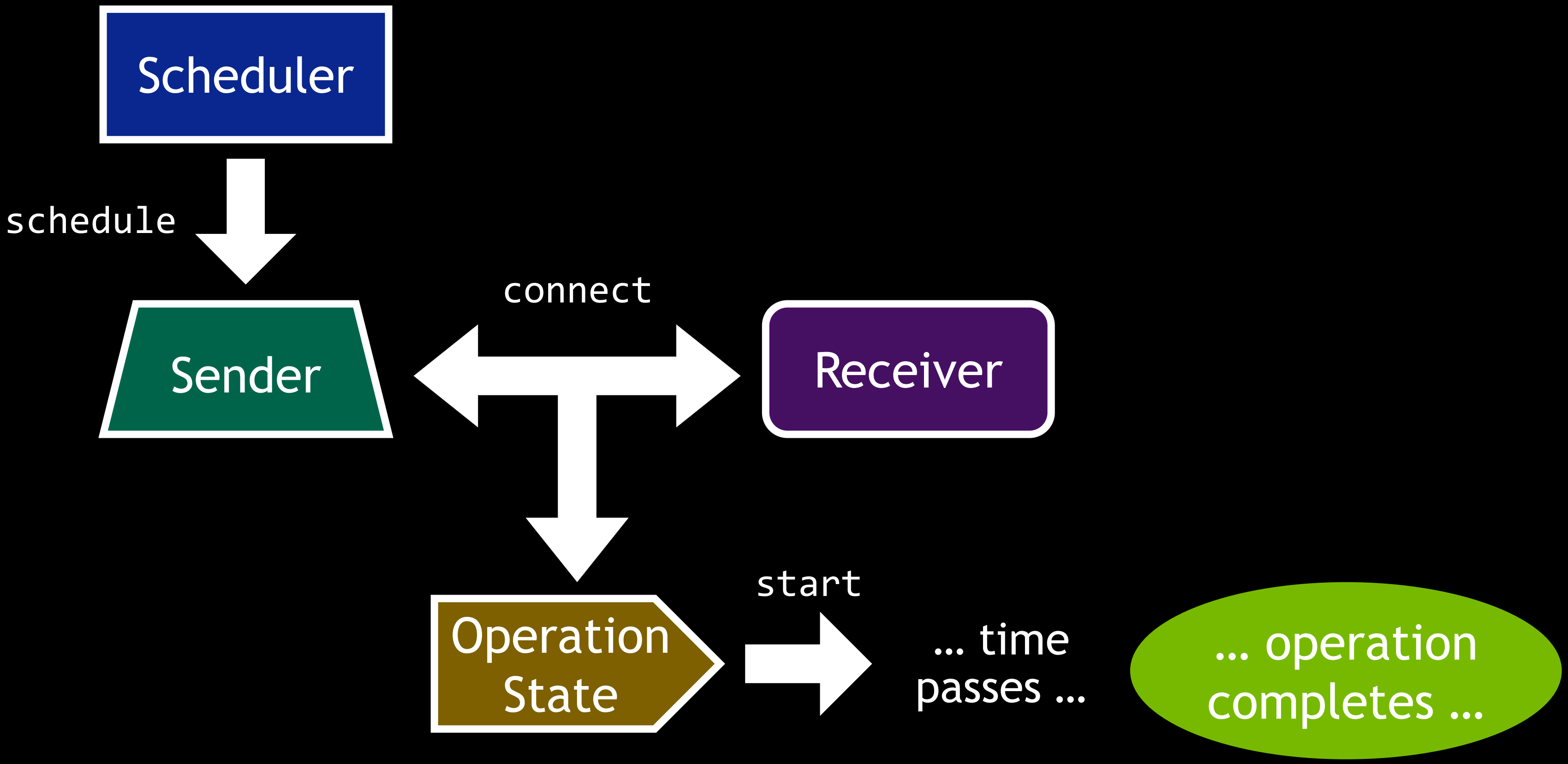
#include <C++>

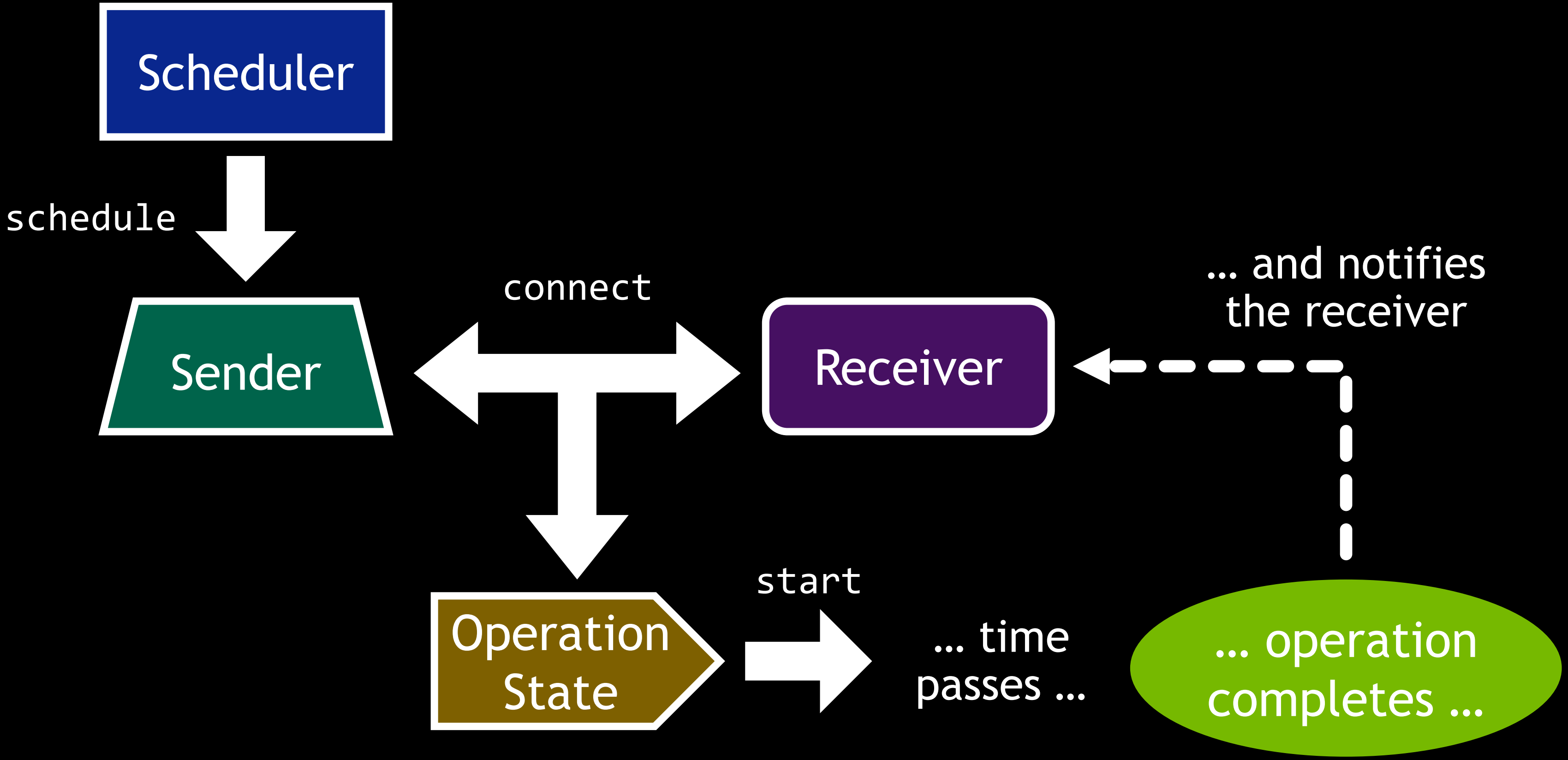

Receiver
set_value(*values...*)
set_error(*error*)
set_done()


Scheduler
schedule
Sender


Scheduler
schedule
Sender
connect
Receiver
Operation State


Scheduler
schedule
connect
Sender
Receiver
Operation State
start


Scheduler
schedule
Sender
connect
Receiver
Operation State
start
… time passes …
… operation completes …


Scheduler
schedule
Sender
connect
Receiver
… and notifies
the receiver
Operation
State
start
… time
passes …
… operation
completes …

```
sender auto f(sender auto p, ...);
```

```
sender auto f(sender auto p, ...);
```

- Takes one or more senders.

```
sender auto f(sender auto p, ...);
```

- Takes one or more senders.
- Return a sender.

```
sender auto f(sender auto p, ...);
```

- Takes one or more senders.
- Return a sender.
- Pipeable (think *nix shells):

  ```
  snd | f | g
  ```

  is equivalent to

  ```
  g(f(snd))
  ```

```
std::vector<std::string_view> v{…};

ex::sender auto s = for_each_async(
                      ex::transfer(
                        unique_async(
                          sort_async(
                            ex::transfer_just(gpu_stream_scheduler{}, v)
                          )
                        ),
                        thread_pool.scheduler()
                      ),
                      [] (std::string_view e)
                      { std::print(file, "{}\n", e); }
                    );

this_thread::sync_wait(s);
```

```
std::vector<std::string_view> v{…};

ex::sender auto s0 = ex::transfer_just(gpu_stream_scheduler{}, v);
ex::sender auto s1 = sort_async(s0);
ex::sender auto s2 = unique_async(s1);
ex::sender auto s3 = ex::transfer(s2, thread_pool.scheduler());
ex::sender auto s4 = for_each_async(s3, [] (std::string_view e)
                                         { std::print(file, "{}\n", e); });

this_thread::sync_wait(s);
```

```
std::vector<std::string_view> v{…};

ex::sender auto s = ex::transfer_just(gpu_stream_scheduler{}, v)
                  | sort_async
                  | unique_async
                  | ex::transfer(thread_pool.scheduler())
                  | for_each_async([] (std::string_view e)
                                   { std::print(file, "{}\n", e); });

this_thread::sync_wait(s);
```

Sender Adaptor	Semantics Of Returned Sender

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<u>then</u>`(sender auto last, invocable auto f)`	Call `f` with the value sent by `last`.

Sender Adaptor	Semantics Of Returned Sender
<u>then</u>`(sender auto last, invocable auto f)`	Call `f` with the value sent by `last`.
<u>bulk</u>`(sender auto last, shape auto n, invocable auto body)`	Call body for every index in n with the value sent by `last`.

Sender Adaptor	Semantics Of Returned Sender
then(sender auto last, invocable auto f)	Call f with the value sent by last.
bulk(sender auto last, shape auto n, invocable auto body)	Call body for every index in n with the value sent by last.
transfer(sender auto last, scheduler auto sch)	Transition to sch for the next sender.

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`bulk(sender auto last, shape auto n, invocable auto body)`	Call body for every index in n with the value sent by `last`.
`transfer(sender auto last, scheduler auto sch)`	Transition to `sch` for the next sender.
`split(sender auto last)`	Can be connected to multiple receivers.

Sender Adaptor	Semantics Of Returned Sender
<u>then</u>`(sender auto last, invocable auto f)`	Call `f` with the value sent by `last`.
<u>bulk</u>`(sender auto last, shape auto n, invocable auto body)`	Call body for every index in n with the value sent by `last`.
<u>transfer</u>`(sender auto last, scheduler auto sch)`	Transition to `sch` for the next sender.
<u>split</u>`(sender auto last)`	Can be connected multiple times.
<u>when_all</u>`(sender auto... inputs)`	Combines multiple senders into an aggregate.

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`then(sender auto last, invocable auto f)`	Call `f` with the value sent by `last`.
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`split(sender auto last)`	Can be connected multiple times.
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`ensure_started(sender auto last)`	Connects and starts `last`.

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Sender Factories	Semantics Of Returned Sender

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`schedule(scheduler auto sch)`	Completes on `sch`.

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Sender Factories	Semantics Of Returned Sender
`schedule(scheduler auto sch)`	Completes on `sch`.
`just(T&&... ts)`	Send the values `ts`.

Sender Adaptor	Semantics Of Returned Sender
then(sender auto last, invocable auto f)	Call f with the value sent by last.
bulk(sender auto last, shape auto n, invocable auto body)	Call body for every index in n with the value sent by last.
transfer(sender auto last, scheduler auto sch)	Transition to sch for the next sender.
split(sender auto last)	Can be connected multiple times.
when_all(sender auto... inputs)	Combines multiple senders into an aggregate.
ensure_started(sender auto last)	Connects and starts last.

Sender Factories	Semantics Of Returned Sender
schedule(scheduler auto sch)	Completes on sch.
just(T&&... ts)	Send the values ts.

Sender Consumers	Semantics

Sender Adaptor	Semantics Of Returned Sender
`then(sender auto last, invocable auto f)`	Call `f` with the value sent by `last`.
`bulk(sender auto last, shape auto n, invocable auto body)`	Call `body` for every index in `n` with the value sent by `last`.
`transfer(sender auto last, scheduler auto sch)`	Transition to `sch` for the next sender.
`split(sender auto last)`	Can be connected multiple times.
`when_all(sender auto... inputs)`	Combines multiple senders into an aggregate.
`ensure_started(sender auto last)`	Connects and starts `last`.

Sender Factories	Semantics Of Returned Sender
`schedule(scheduler auto sch)`	Completes on `sch`.
`just(T&&... ts)`	Send the values `ts`.

Sender Consumers	Semantics
`sync_wait(sender auto snd) -> `*`values-sent-by-sender`*	Block until `snd` completes and return or throw whatever it sent.

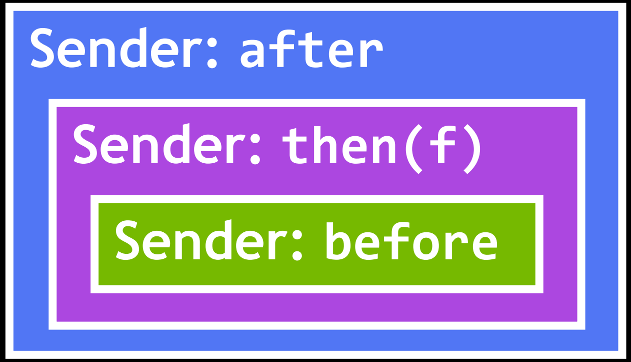
```
before | then(f) | after;
```

```
before | then(f) | after;
```

```
sender auto before_snd = …;
sender auto then_f_snd = then_sender(before_snd, f);
sender auto after_snd  = after_sender(then_f_snd);
```

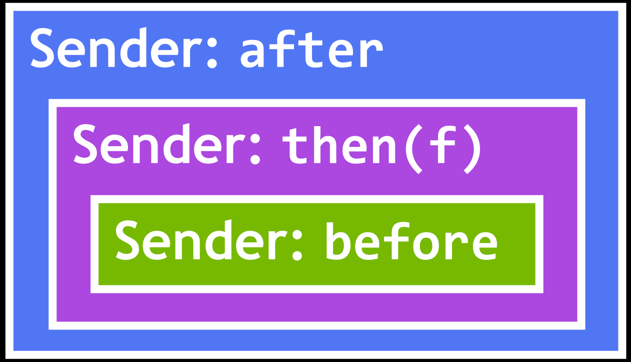
```
before | then(f) | after;
```

```
sender auto before_snd = …;
sender auto then_f_snd = then_sender(before_snd, f);
sender auto after_snd  = after_sender(then_f_snd);
```



```
before | then(f) | after;
```

```
sender auto before_snd = …;
sender auto then_f_snd = then_sender(before_snd, f);
sender auto after_snd  = after_sender(then_f_snd);
 …
     return connect(after_snd, …);
         return connect(then_f_snd, after_rcv);
             return connect(before_snd, then_f_rcv);
                 …
```



before | then(f) | after;

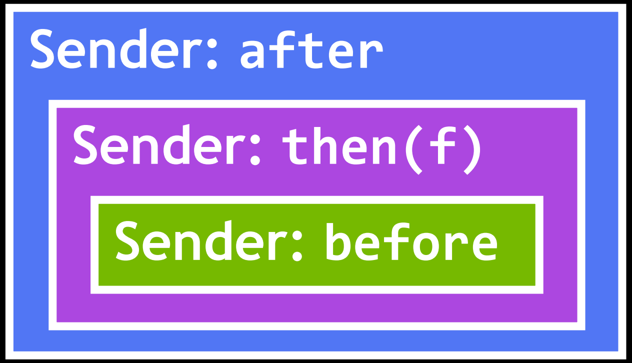
```
sender auto before_snd = …;
sender auto then_f_snd = then_sender(before_snd, f);
sender auto after_snd  = after_sender(then_f_snd);

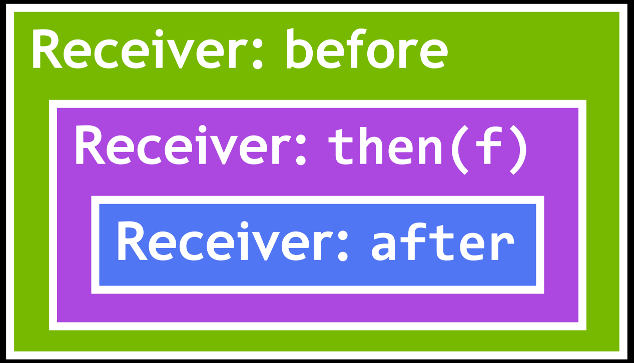
 …
     return connect(after_snd, …);
         return connect(then_f_snd, after_rcv);
             return connect(before_snd, then_f_rcv);
                …
```

before | then(f) | after;

```
sender auto before_snd = …;
sender auto then_f_snd = then_sender(before_snd, f);
sender auto after_snd  = after_sender(then_f_snd);

 …
        return connect(after_snd, …);
               return connect(then_f_snd, after_rcv);
                      return connect(before_snd, then_f_rcv);
                             …
```






```
  …
       set_value(before_rcv, …);
              set_value(then_f_rcv, before_val);
                     set_value(after_rcv, f(before_val));
                            …
```

```
inline constexpr sender_adaptor auto
inclusive_scan_async = [] (…) -> ex::sender auto {
  …
}
```

```
inline constexpr sender_adaptor auto
inclusive_scan_async = [] (ex::sender auto last, auto init, std::size_t tile_count) -> ex::sender auto {
  …
}
```

```
inline constexpr sender_adaptor auto
inclusive_scan_async = [] (ex::sender auto last, auto init, std::size_t tile_count) -> ex::sender auto {
  return last
       | ex::then([=] (stdr::random_access_range auto input) {
                       …
                     })
  …
}
```

```
inline constexpr sender_adaptor auto
inclusive_scan_async = [] (ex::sender auto last, auto init, std::size_t tile_count) -> ex::sender auto {
  return last
       | ex::then([=] (stdr::random_access_range auto input) {
                      std::vector<stdr::range_value_t<decltype(input)>> partials(tile_count + 1);
                      partials[0] = init;
                      return send_values(input, std::move(partials));
                    })
  …
}
```

#include <C++>

```
inline constexpr sender_adaptor auto
inclusive_scan_async = [] (ex::sender auto last, auto init, std::size_t tile_count) -> ex::sender auto {
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  …
}
```

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                    partials[0] = init;
                    return send_values(input, std::move(partials));
                  })
       | ex::bulk(tile_count,
                  [=] (std::size_t i, auto input, auto partials) {
                    …
                  })
  …
}
```

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inline constexpr sender_adaptor auto
inclusive_scan_async = [] (ex::sender auto last, auto init, std::size_t tile_count) -> ex::sender auto {
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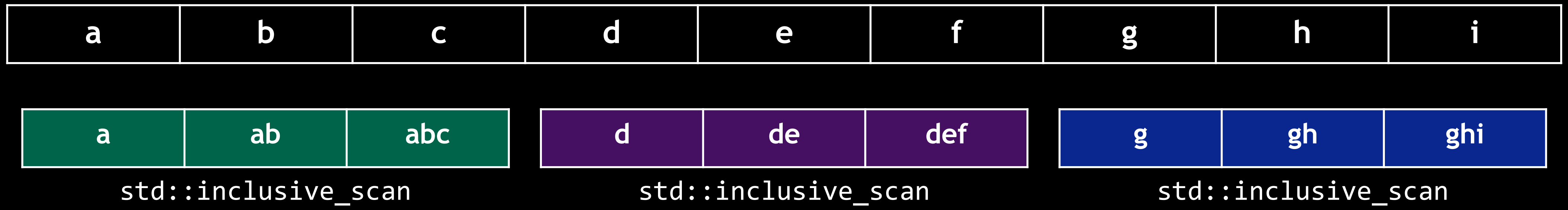
                    std::vector<stdr::range_value_t<decltype(input)>> partials(tile_count + 1);
                    partials[0] = init;
                    return send_values(input, std::move(partials));
                  })
       | ex::bulk(tile_count,
                  [=] (std::size_t i, auto input, auto partials) {
                    auto tile_size  = (input.size() + tile_count - 1) / tile_count;
                    auto start      = i * tile_size;
                    auto end        = std::min(input.size(), (i + 1) * tile_size);
                    …
                  })
  …
}
```

a	b	c	d	e	f	g	h	i

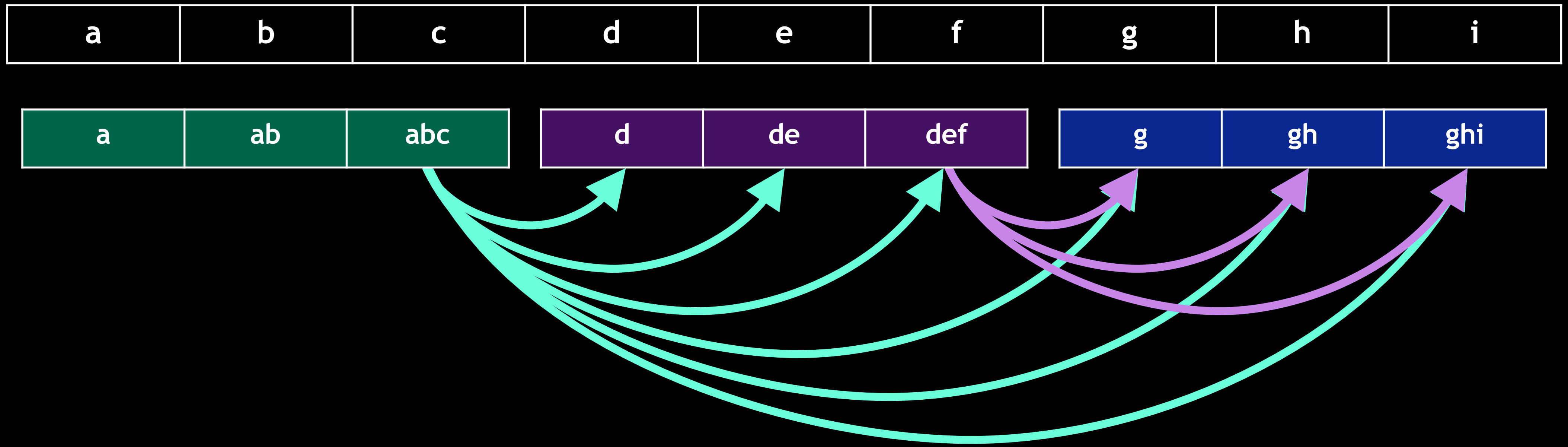
a	ab	abc	d	de	def	g	gh	ghi


a
b
c
d
e
f
g
h
i
a
ab
abc
d
de
def
g
gh
ghi
std::inclusive_scan
std::inclusive_scan
std::inclusive_scan

```
inline constexpr sender_adaptor auto
inclusive_scan_async = [] (ex::sender auto last, auto init, std::size_t tile_count) -> ex::sender auto {
  return last
       | ex::then([=] (stdr::random_access_range auto input) {
                    std::vector<stdr::range_value_t<decltype(input)>> partials(tile_count + 1);
                    partials[0] = init;
                    return send_values(input, std::move(partials));
                  })
       | ex::bulk(tile_count,
                  [=] (std::size_t i, auto input, auto partials) {
                    auto tile_size  = (input.size() + tile_count - 1) / tile_count;
                    auto start      = i * tile_size;
                    auto end        = std::min(input.size(), (i + 1) * tile_size);
                    …                    std::inclusive_scan(begin(input) + start,
                                                             begin(input) + end,
                                                             begin(input) + start);

                  })
  …
}
```

#include <C++>


a
b
c
d
e
f
g
h
i
a
ab
abc
d
de
def
g
gh
ghi

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inline constexpr sender_adaptor auto
inclusive_scan_async = [] (ex::sender auto last, auto init, std::size_t tile_count) -> ex::sender auto {
  return last
       | ex::then([=] (stdr::random_access_range auto input) {
                    std::vector<stdr::range_value_t<decltype(input)>> partials(tile_count + 1);
                    partials[0] = init;
                    return send_values(input, std::move(partials));
                  })
       | ex::bulk(tile_count,
                  [=] (std::size_t i, auto input, auto partials) {
                    auto tile_size  = (input.size() + tile_count - 1) / tile_count;
                    auto start      = i * tile_size;
                    auto end        = std::min(input.size(), (i + 1) * tile_size);
                    …               = *--std::inclusive_scan(begin(input) + start,
                                                              begin(input) + end,
                                                              begin(input) + start);
                  })
  …
}
```

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inline constexpr sender_adaptor auto
inclusive_scan_async = [] (ex::sender auto last, auto init, std::size_t tile_count) -> ex::sender auto {
  return last
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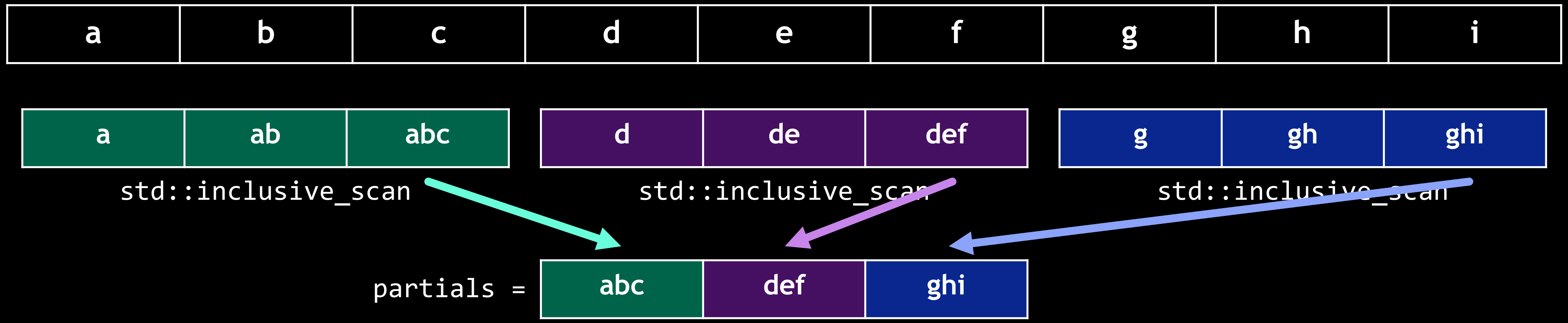
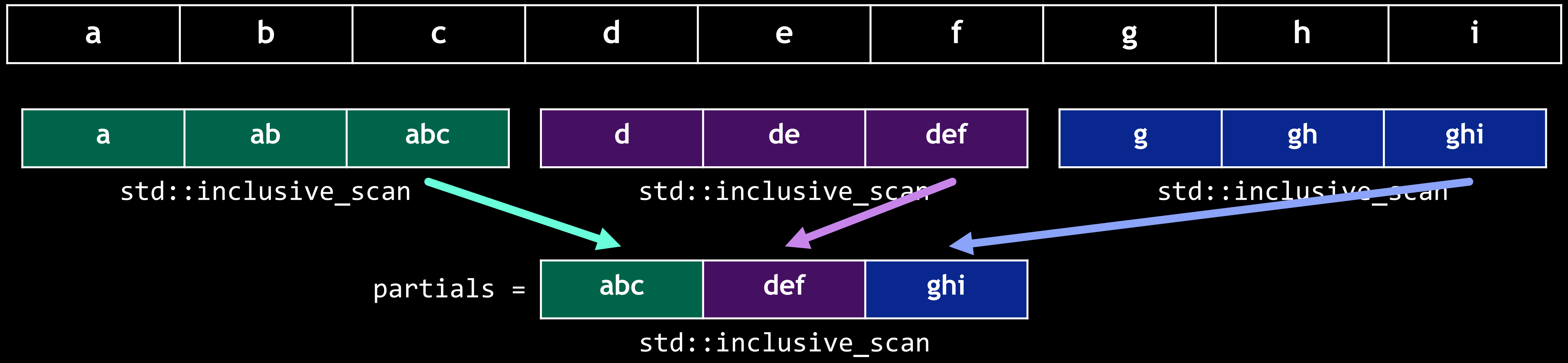
                      auto tile_size  = (input.size() + tile_count - 1) / tile_count;
                      auto start      = i * tile_size;
                      auto end        = std::min(input.size(), (i + 1) * tile_size);
                      partials[i + 1] = *--std::inclusive_scan(begin(input) + start,
                                                               begin(input) + end,
                                                               begin(input) + start);

                    })
  …
}
```


a
b
c
d
e
f
g
h
i
a
ab
abc
d
de
def
g
gh
ghi
std::inclusive_scan
std::inclusive_scan
std::inclusive_scan
partials =
abc
def
ghi


a
b
c
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g
h
i
a
ab
abc
d
de
def
g
gh
ghi
std::inclusive_scan
std::inclusive_scan
std::inclusive_scan
partials =
abc
def
ghi
std::inclusive_scan

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inclusive_scan_async = [] (ex::sender auto last, auto init, std::size_t tile_count) -> ex::sender auto {
  return last
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                    std::vector<stdr::range_value_t<decltype(input)>> partials(tile_count + 1);
                    partials[0] = init;
                    return send_values(input, std::move(partials));
                  })
       | ex::bulk(tile_count,
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                    auto tile_size  = (input.size() + tile_count - 1) / tile_count;
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                    auto end        = std::min(input.size(), (i + 1) * tile_size);
                    partials[i + 1] = *--std::inclusive_scan(begin(input) + start,
                                                             begin(input) + end,
                                                             begin(input) + start);
                  })
       | ex::then([] (auto input, auto partials) {
                    std::inclusive_scan(begin(partials), end(partials), begin(partials));
                    …
                  })
  …
}
```

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inline constexpr sender_adaptor auto
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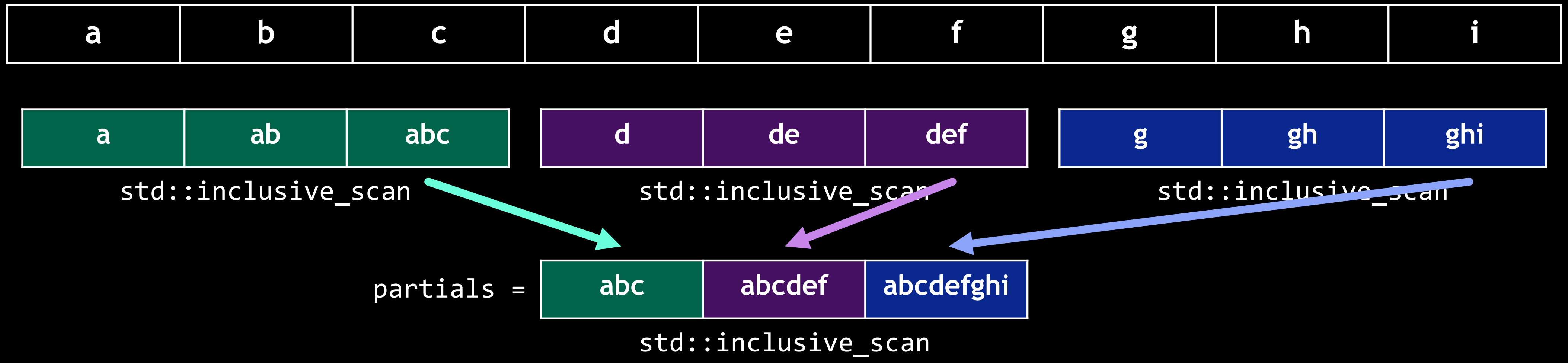
                                                            begin(input) + start);
                 })
       | ex::then([] (auto input, auto partials) {
                   std::inclusive_scan(begin(partials), end(partials), begin(partials));
                   return send_values(input, std::move(partials));
                 })
  …
}
```

#include <C++>



a
b
c
d
e
f
g
h
i
a
ab
abc
d
de
def
g
gh
ghi
std::inclusive_scan
std::inclusive_scan
std::inclusive_scan
partials =
abc
abcdef
abcdefghi
std::inclusive_scan

```
inline constexpr sender_adaptor auto
inclusive_scan_async = [] (ex::sender auto last, auto init, std::size_t tile_count) -> ex::sender auto {
  return last
       | ex::then([=] (stdr::random_access_range auto input) {
                   std::vector<stdr::range_value_t<decltype(input)>> partials(tile_count + 1);
                   partials[0] = init;
                   return send_values(input, std::move(partials));
                 })
       | ex::bulk(tile_count,
                 [=] (std::size_t i, auto input, auto partials) {
                   auto tile_size  = (input.size() + tile_count - 1) / tile_count;
                   auto start      = i * tile_size;
                   auto end        = std::min(input.size(), (i + 1) * tile_size);
                   partials[i + 1] = *--std::inclusive_scan(begin(input) + start,
                                                            begin(input) + end,
                                                            begin(input) + start);
                 })
       | ex::then([] (auto input, auto partials) {
                   std::inclusive_scan(begin(partials), end(partials), begin(partials));
                   return send_values(input, std::move(partials));
                 })
       | ex::bulk(tile_count,
                 [=] (std::size_t i, auto input, auto partials) {
                   …
                 })
  …
}
```

#include <C++>

```
inline constexpr sender_adaptor auto
inclusive_scan_async = [] (ex::sender auto last, auto init, std::size_t tile_count) -> ex::sender auto {
  return last
       | ex::then([=] (stdr::random_access_range auto input) {
                      std::vector<stdr::range_value_t<decltype(input)>> partials(tile_count + 1);
                      partials[0] = init;
                      return send_values(input, std::move(partials));
                    })
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                    [=] (std::size_t i, auto input, auto partials) {
                      auto tile_size  = (input.size() + tile_count - 1) / tile_count;
                      auto start      = i * tile_size;
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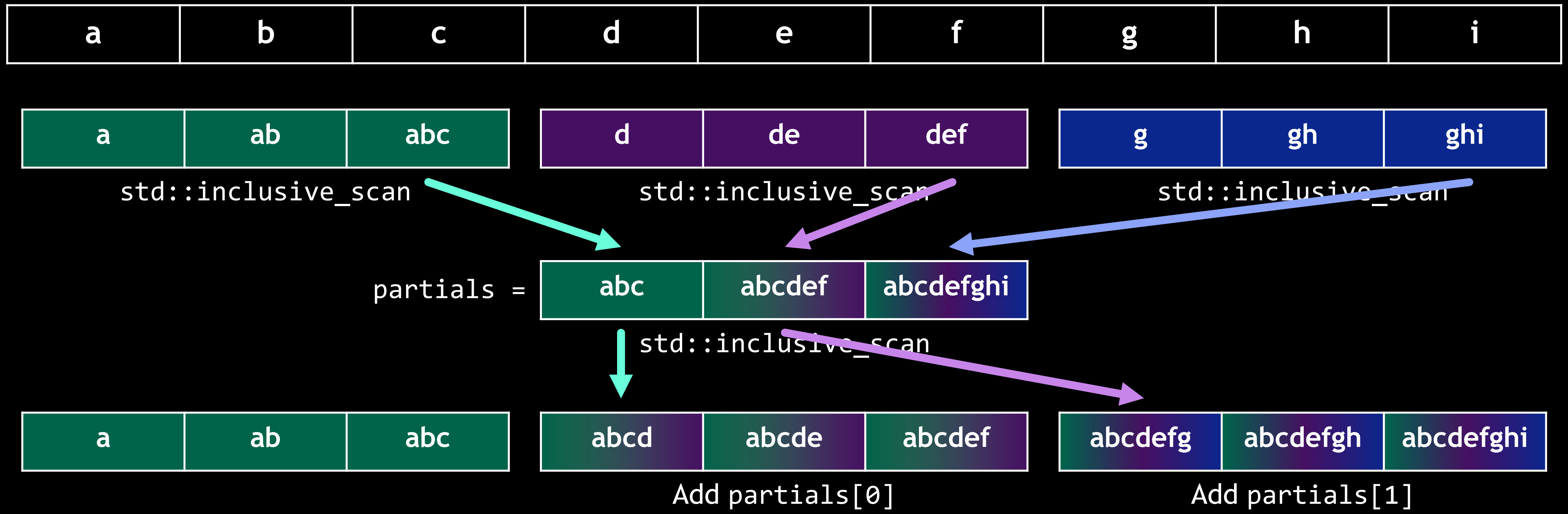
                      partials[i + 1] = *--std::inclusive_scan(begin(input) + start,
                                                               begin(input) + end,
                                                               begin(input) + start);
                    })
       | ex::then([] (auto input, auto partials) {
                      std::inclusive_scan(begin(partials), end(partials), begin(partials));
                      return send_values(input, std::move(partials));
                    })
       | ex::bulk(tile_count,
                    [=] (std::size_t i, auto input, auto partials) {
                      auto tile_size = (input.size() + tile_count - 1) / tile_count;
                      auto start     = i * tile_size;
                      auto end       = std::min(input.size(), (i + 1) * tile_size);
                      …
                    })
  …
}
```

```
inline constexpr sender_adaptor auto
inclusive_scan_async = [] (ex::sender auto last, auto init, std::size_t tile_count) -> ex::sender auto {
  return last
       | ex::then([=] (stdr::random_access_range auto input) {
                    std::vector<stdr::range_value_t<decltype(input)>> partials(tile_count + 1);
                    partials[0] = init;
                    return send_values(input, std::move(partials));
                  })
       | ex::bulk(tile_count,
                  [=] (std::size_t i, auto input, auto partials) {
                    auto tile_size  = (input.size() + tile_count - 1) / tile_count;
                    auto start      = i * tile_size;
                    auto end        = std::min(input.size(), (i + 1) * tile_size);
                    partials[i + 1] = *--std::inclusive_scan(begin(input) + start,
                                                              begin(input) + end,
                                                              begin(input) + start);
                  })
       | ex::then([] (auto input, auto partials) {
                    std::inclusive_scan(begin(partials), end(partials), begin(partials));
                    return send_values(input, std::move(partials));
                  })
       | ex::bulk(tile_count,
                  [=] (std::size_t i, auto input, auto partials) {
                    auto tile_size = (input.size() + tile_count - 1) / tile_count;
                    auto start     = i * tile_size;
                    auto end       = std::min(input.size(), (i + 1) * tile_size);
                    std::for_each(begin(input) + start, begin(input) + end,
                                  [&] (auto& e) { e = partials[i] + e; });
                  })
  …
}
```


a
b
c
d
e
f
g
h
i
a
ab
abc
d
de
def
g
gh
ghi
std::inclusive_scan
std::inclusive_scan
std::inclusive_scan
partials =
abc
abcdef
abcdefghi
std::inclusive_scan
a
ab
abc
abcd
abcde
abcdef
abcdefg
abcdefgh
abcdefghi
Add partials[0]
Add partials[1]

```
inline constexpr sender_adaptor auto
inclusive_scan_async = [] (ex::sender auto last, auto init, std::size_t tile_count) -> ex::sender auto {
  return last
       | ex::then([=] (stdr::random_access_range auto input) {
                    std::vector<stdr::range_value_t<decltype(input)>> partials(tile_count + 1);
                    partials[0] = init;
                    return send_values(input, std::move(partials));
                  })
       | ex::bulk(tile_count,
                  [=] (std::size_t i, auto input, auto partials) {
                    auto tile_size   = (input.size() + tile_count - 1) / tile_count;
                    auto start       = i * tile_size;
                    auto end         = std::min(input.size(), (i + 1) * tile_size);
                    partials[i + 1] = *--std::inclusive_scan(begin(input) + start,
                                                             begin(input) + end,
                                                             begin(input) + start);
                  })
       | ex::then([] (auto input, auto partials) {
                    std::inclusive_scan(begin(partials), end(partials), begin(partials));
                    return send_values(input, std::move(partials));
                  })
       | ex::bulk(tile_count,
                  [=] (std::size_t i, auto input, auto partials) {
                    auto tile_size = (input.size() + tile_count - 1) / tile_count;
                    auto start     = i * tile_size;
                    auto end       = std::min(input.size(), (i + 1) * tile_size);
                    std::for_each(begin(input) + start, begin(input) + end,
                                  [&] (auto& e) { e = partials[i] + e; });
                  })
       | ex::then([=] (auto input, auto partials) { return input; });
}
```

```
inline constexpr sender_adaptor auto
inclusive_scan_async = [] (ex::sender auto last, auto init, std::size_t tile_count) -> ex::sender auto {
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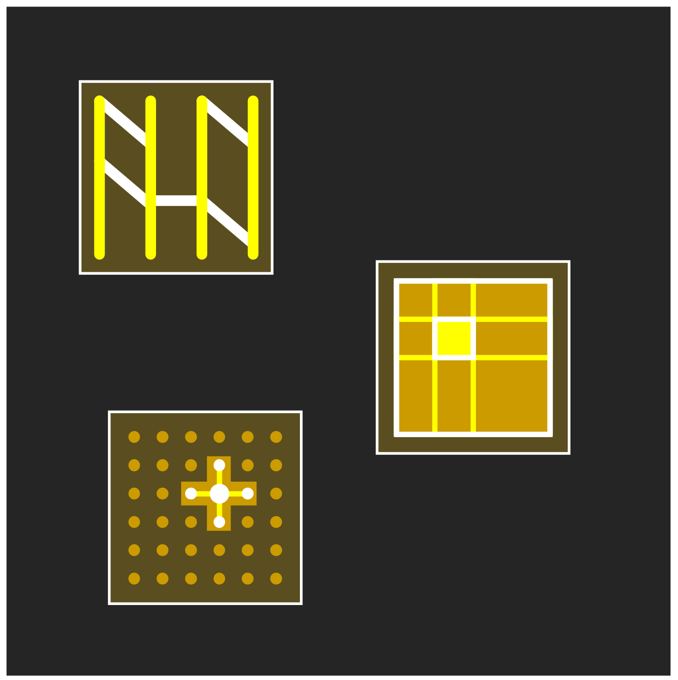
       | ex::then([=] (stdr::random_access_range auto input) {
                   std::vector<stdr::range_value_t<decltype(input)>> partials(tile_count + 1);
                   partials[0] = init;
                   return send_values(input, std::move(partials));
                 })
       | ex::bulk(tile_count,
                 [=] (std::size_t i, auto input, auto partials) {
                   auto tile_size  = (input.size() + tile_count - 1) / tile_count;
                   auto start      = i * tile_size;
                   auto end        = std::min(input.size(), (i + 1) * tile_size);
                   partials[i + 1] = *--std::inclusive_scan(begin(input) + start,
                                                            begin(input) + end,
                                                            begin(input) + start);
                 })
       | ex::then([] (auto input, auto partials) {
                   std::inclusive_scan(begin(partials), end(partials), begin(partials));
                   return send_values(input, std::move(partials));
                 })
       | ex::bulk(tile_count,
                 [=] (std::size_t i, auto input, auto partials) {
                   auto tile_size = (input.size() + tile_count - 1) / tile_count;
                   auto start     = i * tile_size;
                   auto end       = std::min(input.size(), (i + 1) * tile_size);
                   std::for_each(begin(input) + start, begin(input) + end,
                                 [&] (auto& e) { e = partials[i] + e; });
                 })
       | ex::then([=] (auto input, auto partials) { return input; });
}
```

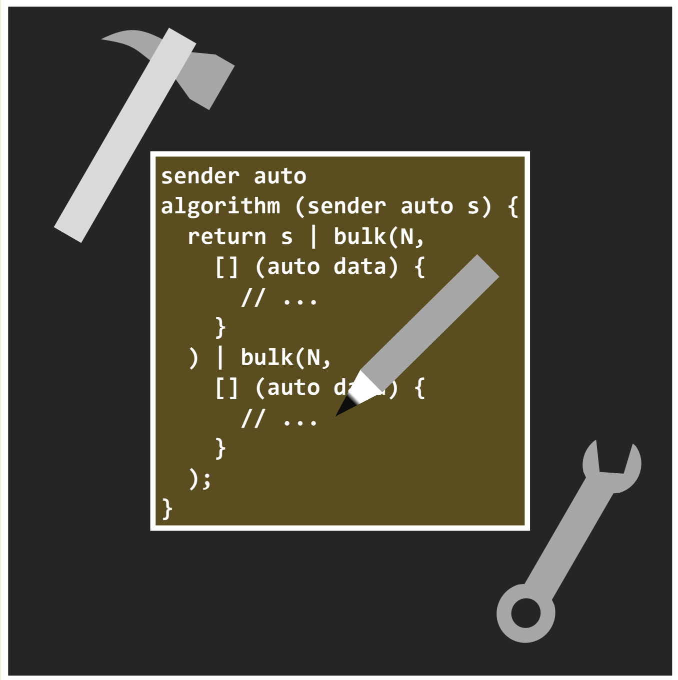
Pillars of C++ Standard Parallelism

Common Algorithms that Dispatch to Vendor-Optimized Parallel Libraries

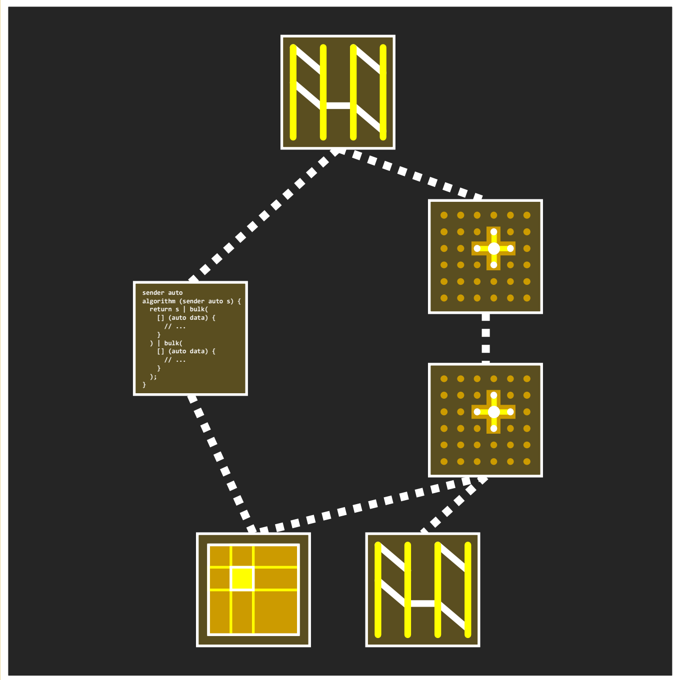


Today

Tools to Write Your Own Parallel Algorithms that Run Anywhere



Mechanisms for Composing Parallel Invocations into Task Graphs



With Senders & Receivers

Standard Algorithms

Serial (C++98)

```
std::vector<T> x{…};

std::for_each(
 begin(x), end(x),
 f);

std::for_each(
 begin(x), end(x),
 g);

std::for_each(
 begin(x), end(x),
 h);
```

Parallel (C++17)

```
std::vector<T> x{…};

std::for_each(
 ex::par_unseq,
 begin(x), end(x),
 f);
std::for_each(
 ex::par_unseq,
 begin(x), end(x),
 g);
std::for_each(
 ex::par_unseq,
 begin(x), end(x),
 h);
```

Asynchronous

```
std::vector<T> x(…);

ex::sender auto s
 = ex::transfer_just(sch, x)
 | for_each_async(f)
 | for_each_async(g)
 | for_each_async(h);

this_thread::sync_wait(s);
```

Today, C++ has no reasonable abstraction for multi-dimensional data.

Today, C++ has no reasonable abstraction for multi-dimensional data.

The solution is coming in C++23:

`std::mdspan`

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- Non-owning; pointer + metadata.

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- Non-owning; pointer + metadata.
- Metadata can be dynamic or static.

std::mdspan

- Non-owning; pointer + metadata.
- Metadata can be dynamic or static.
- Parameterizes layout.

std::mdspan

- Non-owning; pointer + metadata.
- Metadata can be dynamic or static.
- Parameterizes layout and access.

```
template <std::size_t... Extents>
class std::extents;
```

```
template <std::size_t... Extents>
class std::extents;

std::extents e0{16, 32};
// Equivalent to:
std::extents<std::dynamic_extent, std::dynamic_extent> e1{16, 32};


e0.rank()     == 2
e0.extent(0) == 16
e0.extent(1) == 32
```

```
template <std::size_t... Extents>
class std::extents;

std::extents e0{16, 32};
// Equivalent to:
std::extents<std::dynamic_extent, std::dynamic_extent> e1{16, 32};
std::dextents<2> e2{16, 32};

e0.rank()     == 2
e0.extent(0) == 16
e0.extent(1) == 32
```

```
template <std::size_t... Extents>
class std::extents;

std::extents e0{16, 32};
// Equivalent to:
std::extents<std::dynamic_extent, std::dynamic_extent> e1{16, 32};
std::dextents<2> e2{16, 32};

e0.rank()      == 2
e0.extent(0) == 16
e0.extent(1) == 32

std::extents<16, 32> e3;
```

```
template <std::size_t... Extents>
class std::extents;

std::extents e0{16, 32};
// Equivalent to:
std::extents<std::dynamic_extent, std::dynamic_extent> e1{16, 32};
std::dextents<2> e2{16, 32};

e0.rank()      == 2
e0.extent(0) == 16
e0.extent(1) == 32

std::extents<16, 32> e3;

std::extents<16, std::dynamic_extent> e4{32};
```

```
template <std::size_t... Extents>
class std::extents;

std::extents e0{16, 32};
// Equivalent to:
std::extents<std::dynamic_extent, std::dynamic_extent> e1{16, 32};
std::dextents<2> e2{16, 32};

e0.rank()     == 2
e0.extent(0) == 16
e0.extent(1) == 32

std::extents<16, 32> e3;

std::extents<16, std::dynamic_extent> e4{32};

std::extents e5{16, 32, 48, 4};
```

```
template <

                                              >
class std::mdspan;
```

```
template <class T,

                                                  >
class std::mdspan;
```

```
template <class T,
          class Extents,


                                                         >
class std::mdspan;
```

```
template <class T,
          class Extents,
          class LayoutPolicy = std::layout_right,
                                                        >
class std::mdspan;
```

```
template <class T,
          class Extents,
          class LayoutPolicy = std::layout_right,
          class AccessorPolicy = std::default_accessor<T>>
class std::mdspan;
```

```
template <class T,
          class Extents,
          class LayoutPolicy = std::layout_right,
          class AccessorPolicy = std::default_accessor<T>>
class std::mdspan;

std::mdspan m0{data, 16, 32};
// Equivalent to:
std::mdspan<double, std::dextents<2>> m1{data, 16, 32};
```

```
template <class T,
          class Extents,
          class LayoutPolicy = std::layout_right,
          class AccessorPolicy = std::default_accessor<T>>
class std::mdspan;

std::mdspan m0{data, 16, 32};
// Equivalent to:
std::mdspan<double, std::dextents<2>> m1{data, 16, 32};

m0[i, j] == data[i * M + j]
```

```
template <class T,
          class Extents,
          class LayoutPolicy = std::layout_right,
          class AccessorPolicy = std::default_accessor<T>>
class std::mdspan;

std::mdspan m0{data, 16, 32};
// Equivalent to:
std::mdspan<double, std::dextents<2>> m1{data, 16, 32};

m0[i, j] == data[i * M + j]

std::mdspan m2{data, std::extents<16, 32>{}};
// Equivalent to:
std::mdspan<double, std::extents<16, 32>> m3{data};

std::mdspan m4{data, std::extents<16, std::dynamic_extent>{32}};
```

Row-Major AKA Right

- C++, NumPy (default)
- Rightmost extent is contiguous

```
mdspan A{data, N, M};
mdspan A{data, layout_right::mapping{N, M}};

A[i, j]      == data[i * M + j]
A.stride(0) == M
A.stride(1) == 1
```

Row-Major AKA Right

- C++, NumPy (default)
- Rightmost extent is contiguous

```
mdspan A{data, N, M};
mdspan A{data, layout_right::mapping{N, M}};

A[i, j]      == data[i * M + j]
A.stride(0) == M
A.stride(1) == 1
```

Location	Element
0	a_{11}
1	a_{12}
2	a_{21}
3	a_{22}

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

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- C++, NumPy (default)
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```
mdspan A{data, N, M};
mdspan A{data, layout_right::mapping{N, M}};

A[i, j]      == data[i * M + j]
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```

Location	Element
0	a_{11}
1	a_{12}
2	a_{21}
3	a_{22}

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Column-Major AKA Left

- Fortran, MATLAB
- Leftmost extent is contiguous

```
mdspan B{data, layout_left::mapping{N, M}};

B[i, j]      == data[i + j * N]
B.stride(0) == 1
B.stride(1) == N
```

Row-Major AKA Right

- C++, NumPy (default)
- Rightmost extent is contiguous

```
mdspan A{data, N, M};
mdspan A{data, layout_right::mapping{N, M}};

A[i, j]      == data[i * M + j]
A.stride(0) == M
A.stride(1) == 1
```

Location	Element
0	a_{11}
1	a_{12}
2	a_{21}
3	a_{22}

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

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- Fortran, MATLAB
- Leftmost extent is contiguous

```
mdspan B{data, layout_left::mapping{N, M}};

B[i, j]      == data[i + j * N]
B.stride(0) == 1
B.stride(1) == N
```

Location	Element
0	a_{11}
1	a_{21}
2	a_{12}
3	a_{22}

Row-Major AKA Right

- C++, NumPy (default)
- Rightmost extent is contiguous

```
mdspan A{data, N, M};
mdspan A{data, layout_right::mapping{N, M}};

A[i, j]      == data[i * M + j]
A.stride(0) == M
A.stride(1) == 1
```

Column-Major AKA Left

- Fortran, MATLAB
- Leftmost extent is contiguous

```
mdspan B{data, layout_left::mapping{N, M}};

B[i, j]      == data[i + j * N]
B.stride(0) == 1
B.stride(1) == N
```

User-Defined Strides

```
mdspan C{data, layout_stride::mapping{extents{N, M}, {X, Y}}};

A[i, j]      == data[i * X + j * Y]
A.stride(0) == X
A.stride(1) == Y
```

Layouts map $(i, j, k, \dots)$ to a data location.

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Anyone can define a layout.

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Anyone can define a layout.

Layouts may:

- Be non-contiguous.

Layouts map $(i, j, k, \dots)$ to a data location.

Anyone can define a layout.

Layouts may:

- Be non-contiguous.
- Map multiple indices to the same location.

Layouts map $(i, j, k, \ldots)$ to a data location.

Anyone can define a layout.

Layouts may:

- Be non-contiguous.
- Map multiple indices to the same location.
- Perform complicated computations.

Layouts map $(i, j, k, \ldots)$ to a data location.

Anyone can define a layout.

Layouts may:

- Be non-contiguous.
- Map multiple indices to the same location.
- Perform complicated computations.
- Have or refer to state.

Parametric layout enables generic multi-dimensional algorithms.

```
void your_function(Eigen::Matrix<double, Eigen::Dynamic, Eigen::Dynamic>& m);
```

```
void your_function(Eigen::Matrix<double, Eigen::Dynamic, Eigen::Dynamic>& m);

your_function(Eigen::Matrix<double, Eigen::Dynamic, Eigen::Dynamic>{…});
```

```
void your_function(Eigen::Matrix<double, Eigen::Dynamic, Eigen::Dynamic>& m);

your_function(Eigen::Matrix<double, Eigen::Dynamic, Eigen::Dynamic>{…});
your_function(boost::numeric::ublas::matrix<double>{…});
your_function(Mat{…}); // PETSc
your_function(blaze::DynamicMatrix<double, blaze::rowMajor>{…});
your_function(cutlass::HostTensor<float, cutlass::layout::ColumnMajor>{…});
// …
```

```
void your_function(std::mdspan<T, Extents, Layout, Accessor> m);

your_function(Eigen::Matrix<double, Eigen::Dynamic, Eigen::Dynamic>{…});
your_function(boost::numeric::ublas::matrix<double>{…});
your_function(Mat{…}); // PETSc
your_function(blaze::DynamicMatrix<double, blaze::rowMajor>{…});
your_function(cutlass::HostTensor<float, cutlass::layout::ColumnMajor>{…});
// …
```

```
struct my_matrix {
public:
  my_matrix(std::size_t N, std::size_t M)
    : num_rows_(N), num_cols_(M), storage_(num_rows_ * num_cols_) {}

  double& operator()(size_t i, size_t j)
  { return storage_[i * num_cols_ + j]; }
  const double& operator()(size_t i, size_t j) const
  { return storage_[i * num_cols_ + j]; }

  std::size_t num_rows() const { return num_rows_; }
  std::size_t num_cols() const { return num_cols_; }




private:
  std::size_t num_rows_, num_cols_;
  std::vector<double> storage_;
};
```

```
struct my_matrix {
public:
  my_matrix(std::size_t N, std::size_t M)
    : num_rows_(N), num_cols_(M), storage_(num_rows_ * num_cols_) {}

  double& operator()(size_t i, size_t j)
  { return storage_[i * num_cols_ + j]; }
  const double& operator()(size_t i, size_t j) const
  { return storage_[i * num_cols_ + j]; }

  std::size_t num_rows() const { return num_rows_; }
  std::size_t num_cols() const { return num_cols_; }

  operator std::mdspan<double, std::dextents<2>> const
  { return {storage_, num_rows_, num_cols_}; }

private:
  std::size_t num_rows_, num_cols_;
  std::vector<double> storage_;
};
```

```
std::mdspan A{input,  N, M, O};

std::mdspan B{output, N, M, O};



auto v = stdv::cartesian_product(
  stdv::iota(1, A.extent(0) - 1),
  stdv::iota(1, A.extent(1) - 1),
  stdv::iota(1, A.extent(2) - 1));

std::for_each(ex::par_unseq,
  begin(v), end(v),
  [=] (auto idx) {
    auto [i, j, k] = idx;
    B[i, j, k] =   ( A[i, j, k-1] +
                     A[i-1, j, k] +
      A[i, j-1, k] + A[i, j,   k] + A[i, j+1, k]
                   + A[i+1, j, k]
                   + A[i, j, k+1] ) / 7.0
  });
```

```
std::mdspan A{input,
              std::layout_left::mapping{N, M, O}};
std::mdspan B{output,
              std::layout_left::mapping{N, M, O}};


auto v = stdv::cartesian_product(
  stdv::iota(1, A.extent(0) - 1),
  stdv::iota(1, A.extent(1) - 1),
  stdv::iota(1, A.extent(2) - 1));

std::for_each(ex::par_unseq,
  begin(v), end(v),
  [=] (auto idx) {
    auto [i, j, k] = idx;
    B[i, j, k] =   ( A[i, j, k-1] +
                     A[i-1, j, k] +
      A[i, j-1, k] + A[i, j,   k] + A[i, j+1, k]
                   + A[i+1, j, k]
                   + A[i, j, k+1] ) / 7.0
  });
```

#include <C++>

```
std::span A{input,  N * M};
std::span B{output, M * N};

auto v = stdv::cartesian_product(
  stdv::iota(0, N),
  stdv::iota(0, M));

std::for_each(ex::par_unseq,
  begin(v), end(v),
  [=] (auto idx) {
    auto [i, j] = idx;
    B[i + j * N] = A[i * M + j];
  });
```

```
std::mdspan A{input,  N, M};
std::mdspan B{output, M, N};

auto v = stdv::cartesian_product(
  stdv::iota(0, A.extent(0)),
  stdv::iota(0, A.extent(1)));

std::for_each(ex::par_unseq,
  begin(v), end(v),
  [=] (auto idx) {
    auto [i, j] = idx;
    B[j, i] = A[i, j];
  });
```

```
std::mdspan A{input,  N, M};
std::mdspan B{output, M, N};

stdr::for_each(
  ex::par_unseq,
  A.indices(),
  [=] (auto [i, j]) {
    B[j, i] = A[i, j];
  });
```

```
std::mdspan A{input,  N, M};
std::mdspan B{output, M, N};

ex::sender auto s =
    ex::transfer_just(sch, A.indices())
  | for_each_async(
      [=] (auto [i, j]) {
        B[j, i] = A[i, j];
      });
```

```
submdspan(mdspan<…> m, SliceSpecifiers... ss)
  -> mdspan<…>
```

```
submdspan(mdspan<…> m, SliceSpecifiers... ss)
  -> mdspan<…>
```

Slice Specifier	Argument	Reduces Rank?
Single Index	`Integral`	☑

```
submdspan(mdspan<…> m, SliceSpecifiers... ss)
    -> mdspan<…>
```

Slice Specifier	Argument	Reduces Rank?
Single Index	`Integral`	☑
Range of Indices	`std::pair<Integral, Integral>` `std::tuple<Integral, Integral>`	☒

```
submdspan(mdspan<…> m, SliceSpecifiers... ss)
    -> mdspan<…>
```

Slice Specifier	Argument	Reduces Rank?
Single Index	`Integral`	☑
Range of Indices	`std::pair<Integral, Integral>` `std::tuple<Integral, Integral>`	☒
All Indices	`std::full_extent`	☒

```
std::mdspan m0{64, 128, 32};

auto m1 = std::submdspan(m0, std::tuple{16, 24},
                             std::tuple{32, 40},
                             std::tuple{ 8, 16});
```

```
std::mdspan m0{64, 128, 32};

auto m1 = std::submdspan(m0, std::tuple{16, 24},
                             std::tuple{32, 40},
                             std::tuple{ 8, 16});

m1.rank()    == 3
```

```
std::mdspan m0{64, 128, 32};

auto m1 = std::submdspan(m0, std::tuple{16, 24},
                             std::tuple{32, 40},
                             std::tuple{ 8, 16});

m1.rank()     == 3
m1.extent(0) == 8
m1.extent(1) == 8
m1.extent(2) == 8
```

```
std::mdspan m0{64, 128, 32};

auto m1 = std::submdspan(m0, std::tuple{16, 24},
                             std::tuple{32, 40},
                             std::tuple{ 8, 16});

m1.rank()     == 3
m1.extent(0) == 8
m1.extent(1) == 8
m1.extent(2) == 8
m1[i, j, k]  == m0[i + 16, j + 32, k + 8]
```

```
std::mdspan m0{64, 128, 32};

auto m1 = std::submdspan(m0, std::tuple{16, 24},
                             std::tuple{32, 40},
                             std::tuple{ 8, 16});
m1.rank()     == 3
m1.extent(0) == 8
m1.extent(1) == 8
m1.extent(2) == 8
m1[i, j, k]  == m0[i + 16, j + 32, k + 8]

auto m2 = std::submdspan(m0, 16,
                             std::full_extent,
                             32);
```

```
std::mdspan m0{64, 128, 32};

auto m1 = std::submdspan(m0, std::tuple{16, 24},
                             std::tuple{32, 40},
                             std::tuple{ 8, 16});
m1.rank()     == 3
m1.extent(0) == 8
m1.extent(1) == 8
m1.extent(2) == 8
m1[i, j, k]  == m0[i + 16, j + 32, k + 8]

auto m2 = std::submdspan(m0, 16,
                             std::full_extent,
                             32);
m2.rank() == 1
```

```
std::mdspan m0{64, 128, 32};

auto m1 = std::submdspan(m0, std::tuple{16, 24},
                             std::tuple{32, 40},
                             std::tuple{ 8, 16});
m1.rank()     == 3
m1.extent(0) == 8
m1.extent(1) == 8
m1.extent(2) == 8
m1[i, j, k]  == m0[i + 16, j + 32, k + 8]

auto m2 = std::submdspan(m0, 16,
                             std::full_extent,
                             32);
m2.rank()     == 1
m2.extent(0) == 128
```

```
std::mdspan m0{64, 128, 32};

auto m1 = std::submdspan(m0, std::tuple{16, 24},
                             std::tuple{32, 40},
                             std::tuple{ 8, 16});
m1.rank()     == 3
m1.extent(0) == 8
m1.extent(1) == 8
m1.extent(2) == 8
m1[i, j, k]  == m0[i + 16, j + 32, k + 8]

auto m2 = std::submdspan(m0, 16,
                             std::full_extent,
                             32);
m2.rank()     == 1
m2.extent(0) == 128
m2[j]         == m0[16, j, 32]
```

```
std::mdspan A{input,  N, M};
std::mdspan B{output, M, N};
std::size_t T = ...;
```

#include <C++>

```
std::mdspan A{input,  N, M};
std::mdspan B{output, M, N};
std::size_t T = ...;

auto outer = stdv::cartesian_product(stdv::iota(0, (N + T - 1) / T),
                                     stdv::iota(0, (M + T - 1) / T));
```

```
std::mdspan A{input,  N, M};
std::mdspan B{output, M, N};
std::size_t T = ...;

auto outer = stdv::cartesian_product(stdv::iota(0, (N + T - 1) / T),
                                     stdv::iota(0, (M + T - 1) / T));

std::for_each(ex::par_unseq, begin(outer), end(outer),
  [=] (auto tile) {
    auto [x, y] = tile;
    …
  });
```

```
std::mdspan A{input,  N, M};
std::mdspan B{output, M, N};
std::size_t T = ...;

auto outer = stdv::cartesian_product(stdv::iota(0, (N + T - 1) / T),
                                     stdv::iota(0, (M + T - 1) / T));

std::for_each(ex::par_unseq, begin(outer), end(outer),
  [=] (auto tile) {
    auto [x, y] = tile;
    std::tuple selectN{T * x, std::min(T * (x + 1), N)};
    std::tuple selectM{T * y, std::min(T * (y + 1), M)};

    …
  });
```

```
std::mdspan A{input,  N, M};
std::mdspan B{output, M, N};
std::size_t T = ...;

auto outer = stdv::cartesian_product(stdv::iota(0, (N + T - 1) / T),
                                     stdv::iota(0, (M + T - 1) / T));

std::for_each(ex::par_unseq, begin(outer), end(outer),
  [=] (auto tile) {
    auto [x, y] = tile;
    std::tuple selectN{T * x, std::min(T * (x + 1), N)};
    std::tuple selectM{T * y, std::min(T * (y + 1), M)};

    auto TA = std::submdspan(A, selectN, selectM);
    auto TB = std::submdspan(B, selectM, selectN);

    …
  });
```

```
std::mdspan A{input,  N, M};
std::mdspan B{output, M, N};
std::size_t T = ...;

auto outer = stdv::cartesian_product(stdv::iota(0, (N + T - 1) / T),
                                     stdv::iota(0, (M + T - 1) / T));

std::for_each(ex::par_unseq, begin(outer), end(outer),
  [=] (auto tile) {
    auto [x, y] = tile;
    std::tuple selectN{T * x, std::min(T * (x + 1), N)};
    std::tuple selectM{T * y, std::min(T * (y + 1), M)};

    auto TA = std::submdspan(A, selectN, selectM);
    auto TB = std::submdspan(B, selectM, selectN);

    auto inner = stdv::cartesian_product(stdv::iota(0, TA.extent(0)),
                                         stdv::iota(0, TA.extent(1)));


    …
  });
```

```
std::mdspan A{input,  N, M};
std::mdspan B{output, M, N};
std::size_t T = ...;

auto outer = stdv::cartesian_product(stdv::iota(0, (N + T - 1) / T),
                                     stdv::iota(0, (M + T - 1) / T));

std::for_each(ex::par_unseq, begin(outer), end(outer),
  [=] (auto tile) {
    auto [x, y] = tile;
    std::tuple selectN{T * x, std::min(T * (x + 1), N)};
    std::tuple selectM{T * y, std::min(T * (y + 1), M)};

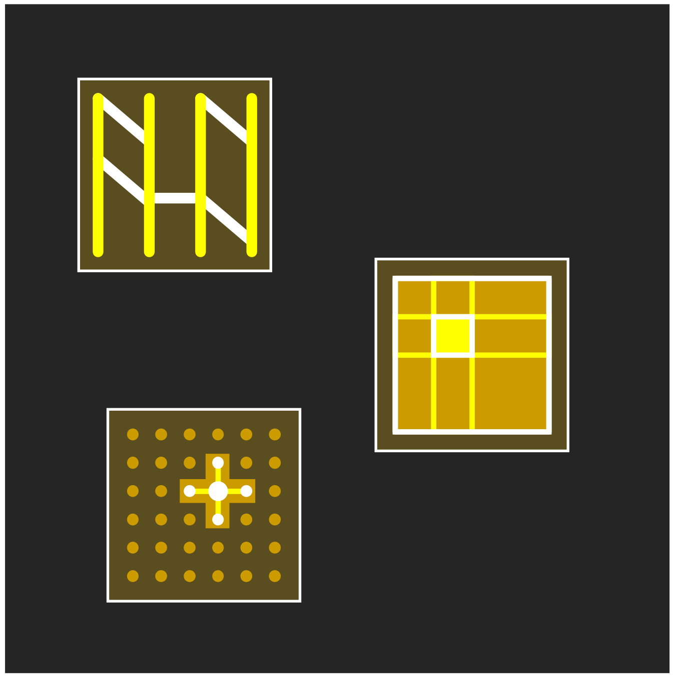
    auto TA = std::submdspan(A, selectN, selectM);
    auto TB = std::submdspan(B, selectM, selectN);

    auto inner = stdv::cartesian_product(stdv::iota(0, TA.extent(0)),
                                         stdv::iota(0, TA.extent(1)));

    for (auto [i, j] : inner)
      TB[j, i] = TA[i, j];
  });
```

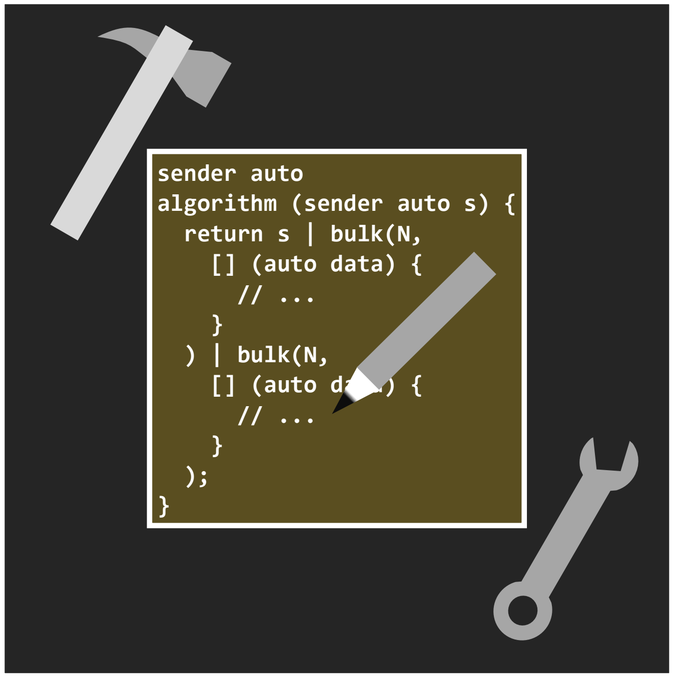
Pillars of C++ Standard Parallelism

Common Algorithms that Dispatch to Vendor-Optimized Parallel Libraries

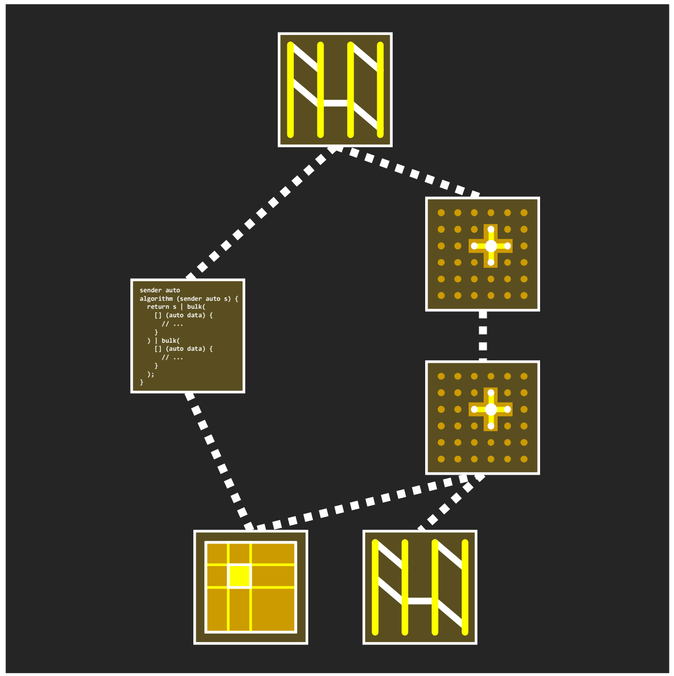


Expanding the Set

Tools to Write Your Own Parallel Algorithms that Run Anywhere



Mechanisms for Composing Parallel Invocations into Task Graphs



```
std::mdspan A{…, N, M};
std::mdspan x{…, M};
std::mdspan y{…, N};

// y = 3.0 A x + 2.0 y
std::matrix_vector_product(
  ex::par_unseq,
  std::scaled(3.0, A), x,
  std::scaled(2.0, y), y);
```

```
std::mdspan A{…, N, M};
std::mdspan x{…, M};
std::mdspan y{…, N};

// y = 3.0 A x + 2.0 y
std::matrix_vector_product(
  ex::par_unseq,
  std::scaled(3.0, A), x,
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std::mdspan A{…, N, M};
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// y = 3.0 A x + 2.0 y
std::matrix_vector_product(
  ex::par_unseq,
  std::scaled(3.0, A), x,
  std::scaled(2.0, y), y);
```

```
std::mdspan A{…, N, M};
std::mdspan x{…, M};
std::mdspan b{…, N};

// Solve A x = b where A = U^T U

// Solve U^T c = b, using x to store c
std::triangular_matrix_vector_solve(ex::par_unseq,
                                    std::transposed(A),
                                    std::upper_triangle, std::explicit_diagonal,
                                    b, x);

// Solve U x = c, overwriting x with result
std::triangular_matrix_vector_solve(ex::par_unseq,
                                    A,
                                    std::upper_triangle, std::explicit_diagonal,
                                    x);
```

```
std::mdspan A{…, N, M};
std::mdspan x{…, M};
std::mdspan b{…, N};

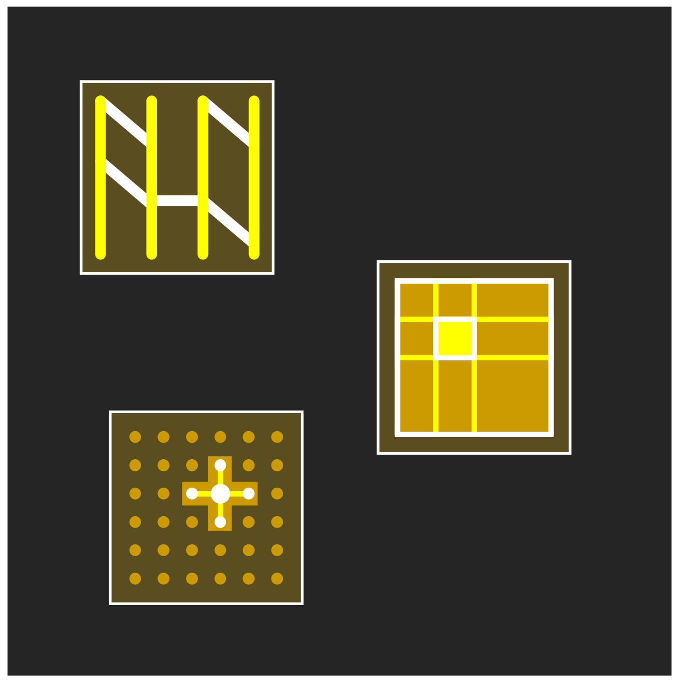
// Solve A x = b where A = U^T U

// Solve U^T c = b, using x to store c
std::triangular_matrix_vector_solve(ex::par_unseq,
                                    std::transposed(A),
                                    std::upper_triangle, std::explicit_diagonal,
                                    b, x);

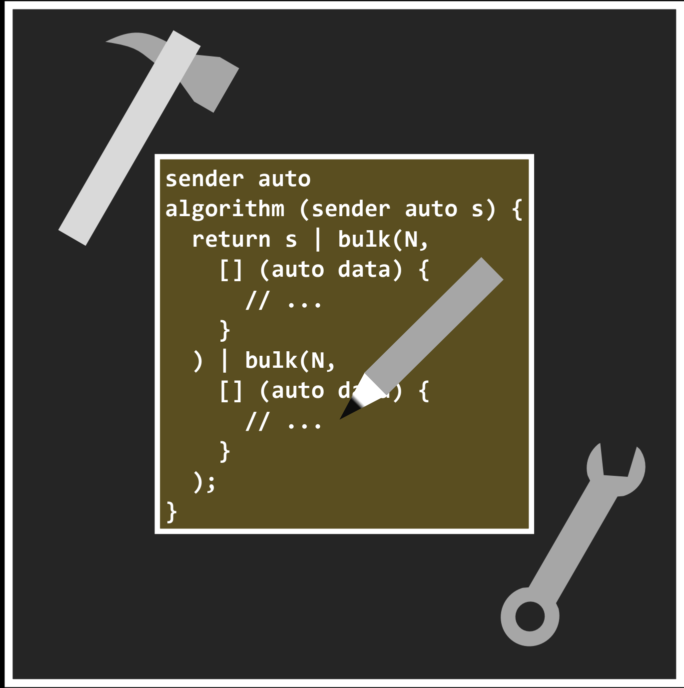
// Solve U x = c, overwriting x with result
std::triangular_matrix_vector_solve(ex::par_unseq,
                                    A,
                                    std::upper_triangle, std::explicit_diagonal,
                                    x);
```

Pillars of C++ Standard Parallelism

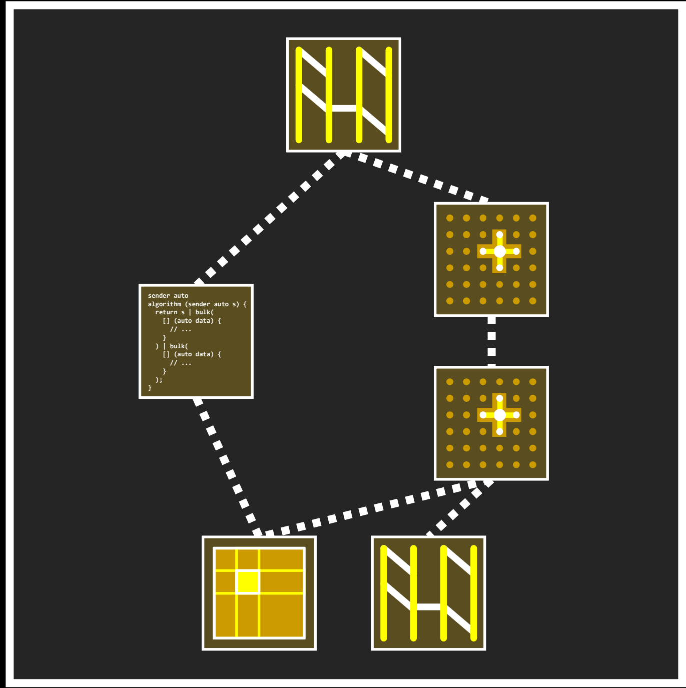
Common Algorithms that Dispatch to Vendor-Optimized Parallel Libraries



Tools to Write Your Own Parallel Algorithms that Run Anywhere



Mechanisms for Composing Parallel Invocations into Task Graphs



We Need On-Ramps